LOGIC

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GROUNDWORK OF LOGIC

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PREFACE.

THE first principle underlying this book is that sound theoretical knowledge is the outcome of reflexion on practical doing, so that, in a sense, every true student makes his science for himself. It, therefore, asks its readers to examine examples of that practical activity of thought with which, as a fact of life, they are familiar, and to lay bare the principles they find operative in them. As in actual thought unsound reasoning is as common as sound reasoning, and as truth is made clearer by comparison with error, much attention is given to mistaken thought, or fallacy. Nor is the dragging into light the intellectual sins that do so easily beset us the least valuable service a study of logic can render. The numerous examples are, therefore, not mere illustrations; they are the selected facts with which the investigation directly deals.

By such a mode of study the student will lay a sure foundation on which any further structure of theoretical elaboration may be securely erected. So, too, he may face with confidence any elementary examination in logic. Always, let it be remembered, the test of progress is not ability to reproduce, but power to solve problems.

But students preparing for examinations are surely not the only people to whom a grasp of the difference between cogent and futile argumentation is of worth. Others may not need to enter into the formal distinctions treated in Chapters V. and XIV., but such an analysis of thought as

occupies the rest of the book should be of interest and value to many. Teachers at universities often lament that want of power, both to originate solutions and critically to weigh evidence, is at once the most common and the most serious of the deficiencies of the students who come to them. Schools often do not act on the truth that not mere practice, but examined and criticised practice, is the road to the acquirement of skill in thinking, as in every other form of human activity. A short course of lessons on logic, combined with the working of exercises, would make apparent to young people of sixteen or seventeen what thinking is, and how it is well done. Nor can it be doubted that physical science with its inferences from laboratory work, history and literature with their appeals to evidence, would be more profitably studied by a student who clearly realises the difference between sound and unsound reasoning than by one who has never given attention to this fundamental distinction.

The most profitable way to study such a book as this is first to read it straight through with care, but without attempting to master every detail. The object of this preliminary reading is to gain a general idea of the whole subject, and of the relations of its parts. Here it would be well to omit Chapters V. and XIV. Then should follow a detailed study, chapter by chapter. After each chapter, knowledge of the theoretical results reached should be tested by the questions, and power of application of theory to practice both tested and developed by working the exercises, at the end of the book The importance of doing this thoroughly cannot be over emphasised. Logic and arithmetic are alike in the relative importance of theoretical discussion and practical application. A student who solves all the exercises will go far in making principles of clear

thinking automatic in his thought. This detailed study should be followed by a final continuous reading, that the parts may be firmly bound together in a coherent system.

The second principle underlying the book is that logic is an organic whole. This accounts for the order in which the topics are treated. The customary arrangement is due partly to historical reasons, partly to pedagogical theory. Induction is made to follow deduction because it was added in the last century to the traditional treatment. But this does not represent its place in the actual work of thought. The pedagogical theory was that knowledge grows in the mind by the accretion of atoms, so that teaching should first give elements and then combine them. It is now recognised that this theory is diametrically opposed to the truth. The assumed primary elements are seen to be the results of long processes of analysis. Neither history nor educational theory, then, justifies adherence to the accustomed order in an introductory book on logic. In the process by which thought reaches knowledge general truths are first sought by induction, then applied by deduction. The ultimate step in analysis, which strips bare the skeleton of a process of thought, legitimately comes last in treatment. This seems clearly to be the right path for beginners, whatever may be the advantages of treating ultimate forms first in an advanced treatise on theoretical logic. To begin with such a treatise is as inept as to begin the study of physics with Newton's *Principia*.

I am indebted to my friend Mr. W. P. Welpton, B.Sc., Lecturer in Education in the University of Leeds, for the suggestion of several examples.

J. W.

July 1917.

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CHAPTER I.

THINKING.

1. Nature of Thinking.

(i) Skill.—There is a great difference between doing a thing and doing it well. Everybody recognises this in many matters. Crowds will pay for admission to a hall to hear a great singer or pianist, or to see the works of a great artist. Men know that singing, playing the piano, and painting, do not come by nature; that they are works of skill, and that skill has to be acquired by much well-directed practice.

It is not so generally acknowledged that skill plays a part in our common daily doings. Yet a little thought convinces us that so it is. We all eat, but some are so unskilful in choice of viands, or in mode of mastication, that they bring on themselves various digestive troubles. We all walk, but in towns the facilities for cheap riding lead people not to practise the art. It results that many are unable to walk even twenty miles in a day. And a little observation as one goes about the streets gives abundant evidence that what is wanting in endurance is not always made up in grace. We all talk, but few talk well. Whether regard be paid to enunciation, to the arrangement of words in sentences, or of sentences into discourse, the general average is low. The daily speech of many, even in the educated classes, is slovenly both in structure and in utterance. Were talking, like singing, a luxury, and not a necessity of intercourse,

people would take as much pains in acquiring the one as the other. After all, both are modes of utterance, designed to affect others.

Wherever there are degrees of excellence in doing things, there is skill. And skill can be cultivated.

(ii) Thinking is a Work of Skill.—And we all think. Here, as elsewhere, degrees of native power are obvious. As few could attain the highest rank in singing or painting or oratory, so few have been the great thinkers of the world. But it is of importance to everybody to train himself to think as well as is possible to him.

Perhaps it is because people so often do not do this, that they call 'thinking' what is not thinking. When we say 'I think so-and-so' we often mean 'I don't know, but I believe so-and-so.' If the belief is based on examination of evidence the process is really thinking. But often it is a mere guess or prompting of hope. 'I think I shall pass the examination' is correct enough if it mean 'I have carefully compared my knowledge with the requirements, and I am satisfied it will meet them.' But if it be only the outcome of a buoyant heart and a lively imagination, no thought in the strict sense of the word is present.

Putting aside the looser applications of the word, let us examine what thinking really is. I have to take a cross-country journey. I consult time-tables, select the appropriate entries, and decide which trains I had best take in order to make my connexions and waste as little time as possible. This is a simple little piece of thinking, and much inconvenience is likely to be the result if it be not well done. Yet we probably all know people who cannot use a 'Bradshaw' intelligently.

If we have to decide on a much more important piece of conduct, the method is the same. We must have a clear idea of what we wish to accomplish, examine all the possible ways of doing what we wish, select those that seem to us the most

direct, and combine the steps in the order in which we will take them.

This is how the architect plans a house, how the novelist constructs his plot, how the chemist guides his researches, how the philosopher seeks to solve the problems of all existence.

It thus appears that the object of thinking is to solve problems, and that its work is planning. The problems which confront us are both practical and theoretical. Any one of us may be more interested in the one or in the other. But we must make use of both. The man whose interests are practical often despises theory. But he does so blindly. He can solve his own practical problems only by making use of the solutions by others of theoretical problems on which practical success depends. Every practical invention is an application of knowledge gained in the study of theoretical problems.

In each men make mistakes. The invention of aeroplanes shows man's power to utilise certain natural forces which he has discovered. Every successful flight shows knowledge of how to use the system of thought embodied in the machine. But every fall marks a failure in one or both of these connected systems. Practical error involves a theoretical element: it is always failure to realise some relation between ourselves and the things we use, or between those things themselves. That is to say, all error is either failure to think at all, or failure to think correctly.

All are aware that knowledge grows. The ignorant child develops into the learned man; the race of savages into a cultivated people. This growth is the result of thinking. Practice in thinking develops power of thinking, and increases the number of things with which thinking can deal.

Thought is not like electric power which can be turned on to do many kinds of work if only the proper machinery be provided. It is a skilful human activity, and all skill is limited

in its range. Both the good cricketer and the good billiard player use arms and hands, but skill in the one game does not imply skill in the other, for the ball with which the skill deals is differently related to the player in each case.

Always skill implies adaptation of our activity to something outside us. So, power to think well about one kind of thing does not imply an equal power to deal with something quite different. If anyone has that skill too, he has acquired it separately.

(iii) FORMAL AND MATERIAL ASPECTS OF THINKING.—If we fix our attention on any special piece of planning, we see at once that two things are necessary. We must have sufficient knowledge to choose our materials well, and sufficient skill to use them well. This brings out the two elements in all thinking. We use material, and according to the kind of material must be the use we make of it. But that use is always to select and arrange, in relation to a definite purpose or object we wish to accomplish.

There is, then, a general resemblance between all pieces of thinking. This is called the *formal* element in thought. Investigation of the nature of thinking aims especially at understanding this common element. At the same time, we must remember that all thinking is a dealing with real problems, so that its form cannot be found apart from some matter.

Whatever the matter, thought aims at understanding it, so that use can be made of it, either by practically doing something or by extending knowledge. It begins by sorting out and arranging parts of our experience. The baby does not distinguish one thing from another, not even his own body from surrounding objects. He just feels pleased or pained, and has a vague desire to remain pleased or to cease to be pained.

We cannot here trace the process by which he gradually comes to recognise a regularity in the succession of these

vague feelings of his, and the dependence of one on another; for instance, that pain results from touching fire and pleasure from sucking sugar.

But here is the material on which his thought begins to work, and all its further efforts are directed to understanding more and more the constant relations that rule his experience, that is, the uniformities that exist in nature. Knowledge could never begin were there no such uniformities. If fire had boiled water one day, and frozen it another, not only could the steam-engine never have been invented, but man could never have learnt to regulate the simplest of his dealings with fire. And so throughout.

(iv) Thought seeks Explanation.—Man's actions assume that such constant relations underlie the infinite variety of the world about him, so that what happens once will happen again in similar circumstances. His thinking is an attempt to learn just what those relations are. When he knows under what conditions a certain result will be attained, he understands that relation sufficiently to make use of it.

If we push the explanation further and further back, asking the reasons for each set of conditions, we find our thought more and more coming to the idea of systematic arrangement of bodies of forces. In each such system the forces are related in constant ways to each other and to the whole.

An aeroplane is one such small system, in which each part is so related to the other parts and to the whole that the machine can fly by the adaptation of the power generated by the engines to the resistance of the air, and its flight be determined by the pilot.

The progress of exact knowledge is the ever-widening recognition of system. The smaller systems are seen to be parts of wider systems. For example, our solar system is now known to be a member of a wider system in which it is but one star among many.

The goal of thought, and the completion of knowledge, would be the recognition of the whole universe of existing things as one systematic whole. That goal is far from being reached, but that we are on the right road is shown by the constant absorption of narrower into wider systems.

2. Postulates of Knowledge.

When we think, therefore, we assume a certain constancy in the things of the world and in their relations to each other and to ourselves. We assume, further, that no piece of knowledge can be inconsistent with any other piece of knowledge; and, finally, that everything in the world is capable of explanation.

As knowledge is the union of the two factors of the material we gather from the world and the thought which finds a meaning in it, these assumptions are made both of the nature of things and of the nature of thought. They may be summed up in four postulates of knowledge, which may also be called principles or laws of thought.

(i) Identity.—That in like cases we may expect like results is assumed in life even before we begin to think. The baby who has derived pleasure from sucking his bottle expects a like experience the next time he receives it. He assumes practically in his actions that the nature of the contents is the same as on prior occasions, that the action of sucking will have the same effect of transferring those contents from the bottle to his mouth, and that his palate will again be gratified.

When we begin to think about things we never question this regularity. It is only as knowledge advances that we see that the constancy of things is not an absolute unalterable identity, but an identity which persists amid change. The old man is identical with himself as a baby, the oak is identical with the acorn from which it sprung. In living beings personal identity consists in the fact that it is

throughout one and the same developing life, seen under many aspects. Even inorganic things are subject to constant change, though it is often so slow as to be imperceptible. But the sea wears away the cliffs in one place, and raises the beach by its deposits in another, so that, for example, several of the old cinque ports are ports no longer. Biology has taught us further that species of animals and plants are ever being modified, and geology that even 'the everlasting hills' are subject to the great law of change.

In the earlier stages of thought change was regarded as exceptional; in our own day it is recognised as the great law of existence, present everywhere and always. This does not destroy, but only modifies, the way in which we understand the constancy of nature.

The traditional statement of the Principle of Identity was 'Everything is what it is,' or symbolically, *A is A*. This really expressed the view that the world consists of independent things of an unchanging nature, and consequently that knowledge would be complete when each such nature could be defined. Now it is seen that the nature of everything is expressed in its relations to other things and that those relations are always changing.

So any symbolic statement of Identity must be such as will cover many assertions about the ever-changing subject; such as *A is B*, *A is C*, etc. But *A* must remain exactly the same throughout any one piece of reasoning—that is, it must refer to exactly the same things—and *B* or *C* must be applicable to those very things.

It is, however, more satisfactory to discard symbols, for they always suggest that we are dealing with mere empty abstractions, and to state the principle in some such form as *Identical assertions can be made in identical cases.* And they can be made, because in identical cases the results are identical.

(ii) Contradiction.—Both the constancy of nature and

the need for consistency of thought make it impossible for us to believe incompatible assertions about the same things or events. So the Principle of Identity is complemented and made more explicit by the Postulate of Consistency, commonly, though not at all happily, named the Principle of Contradiction, for it is the absence of contradiction which it lays down as necessary to correct thought. We may state it in some such terms as *Contradictory assertions about identical cases cannot both be true.* Symbolically this may be put *A cannot both be B and not be B.*

Of course, the matter of which the assertions are made must be identical in the two cases. 'To-day is rainy' and 'To-day is fine' may both be true if the former statement be made on one day, and the latter on another. Or both may be partially true of the same day, in which rainy and bright intervals succeed each other. There is no contradiction unless the assertions of rain and fine weather be made of exactly the same point of time. Then it is obvious.

It is true that many people do hold contradictory opinions. But they do not recognise that they do so, because they hold them apart in their minds and so do not refer them in thought to identical cases. A man often regards actions as legitimate in politics or in business which he condemns in private life. But if once he see that the cases are essentially identical in that they are acts of the same person in relation to his fellows, he cannot continue to justify in the one sphere of conduct what he condemns in the other.

(iii) Excluded Middle.—We assume, then, under Identity that truth always remains true, and under Contradiction that truth and falsity are incompatible. We complete our assumptions as to the nature of truth about things by postulating that there is nothing intermediate between truth and falsity. This is called the Principle of Excluded Middle, because it signals this exclusion of any such third alternative from the possibilities of clear thinking.

While Contradiction makes it explicit that of two contradictory assertions about identical cases one must be false, Excluded Middle brings out the complementary truth that one must be true. An important practical corollary is that an assertion can be proved indirectly by showing that its contradictory is false, or disproved by establishing the truth of its contradictory.

The Principle may be stated in some such form as *Contradictory assertions about identical cases cannot both be false,* or symbolically *A either is B or is not B*. Of course, the reference is as strictly limited as under the Principle of Contradiction. If the *A* is not identical in both cases there is no contradiction, and so the question of the relation of contradictory assertions does not arise.

The principle involves the assumption that to every precise question the true answer is either 'Yes' or 'No.' That answer any one of us may at any time be unable to give, either because our pertinent knowledge is inadequate, or because the question is not put with sufficient definiteness. For example, we may feel unjustified in asserting a particular act to be either right or wrong. It may seem to us right from some points of view and wrong from others. This means that the act is a complex matter, prompted by various motives and leading to many consequences, some of which we feel to be desirable and others undesirable. The rightness or wrongness of the act is not, therefore, simple and absolute, but a balance of many elements. But of every one of these elements the judgement 'right' or 'wrong' can be truly made. Whether we ourselves can make it depends upon the thoroughness of our knowledge.

The principle does not imply that each individual thinker is compelled by the nature of his thought to come to a dogmatic opinion on every subject under the sun, but only that precise knowledge is possible, and that when discovered it will be found to be self-consistent throughout. The only

rational attitude for the individual in the face of many assertions is to preserve an open mind till sufficient evidence to determine his judgement is available. Doubt is as rational in cases in which there is an insufficiency of pertinent evidence as is assurance when such evidence has been found, examined, and weighed.

(iv) Sufficient Reason.—To think of the universe as a systematic unity, in which everything is what it is because of its relations to other things, is to assume that nothing happens by chance, but is the outcome of the constant action of natural forces. Without this assumption we should be engaging in a wild goose chase when we set out to discover why anything has happened. We, therefore, postulate that *There is a sufficient reason for everything that happens.* And as thought aims at grasping the facts of the world it is also implied that *There is a sufficient reason for every true assertion.*

In a way, this is only a special form of the Principle of Identity, for while the latter affirms that identical conditions will always produce identical results, the former reverses this, asserts that everything results from conditions, and implies that if we find the conditions in one case we know them in all identical cases.

To discover the conditions is to find the sufficient reason why the event occurs—its *causa essendi*, or ground of fact. That necessarily is also a sufficient reason for stating that it will occur under those conditions—the *causa cognoscendi*, or ground of knowledge.

But these two do not always coincide. When we know the sequence we may pass from result to conditions as well as from conditions to result. So either may be a *causa cognoscendi* of the other. If I see a boy fall into a river I know that he will get wet, but if I find him dripping by the side of a river I feel fairly confident that he has been in the water, though I have not seen him there.

Even when I have no scientific knowledge of the *causa essendi* experience may furnish a sufficient *causa cognoscendi.* A boy who was quite well before smoking his first cigar, and is sick immediately after, is justified in attributing the change to the smoking. That he perseveres till he can smoke with impunity is due to his enlarging his own experience by that of others and inferring that as they have formed the habit, and find enjoyment in it, so can he.

Taking the principles together it will be seen that they postulate that the world is knowable, because all that happens results from relations in such a way that from the same combination of forces proceed always and inevitably the same results, and that no single result can be produced in any other way. Exact knowledge, or science, is attained when we have so analysed the happenings in the world that we can set before our minds conditions and result free from the admixture of other elements which might obscure the relation.

3. Value of Logic.

(i) Nature of Logic.—Thinking is a work of skill, and there is no progress in any form of skill unless one sees both where and how one's practice is faulty. So only can one try to avoid similar errors in the future, and take means to secure the desired power more completely.

Every executive power which is developed by practice is, however, apt to run into a rut, and become stereotyped by habit. In practical matters we speak of this as 'rule of thumb, and experience shows that while this kind of skill is admirable so long as the task to be done does not vary, it breaks down more or less completely whenever a change in the work requires some adaptation of the activity.

The 'rule of thumb' workman has made his skill as like as possible to the instinctive skill by which bees build their hives. In both cases it is opposed to progress, because it

cannot deal appropriately with new situations. This is the work of thought. It implies seeing in what the novelty consists, and what modification of the customary mode of activity is needed to meet its requirements.

If each one of us seeks in his own life, he will find many cases of the petrification by custom not only of practical activities, but of affections and opinions. This means that, though we *can* think, we do not always think. We think only when something presents itself as a difficulty to be overcome, or a problem to be solved.

If, then, we would obtain skill in solving the problems which life offers us, we must set ourselves to learn the art of thinking. We must make clear to ourselves what distinguishes valid thought from erroneous thinking. This shows both the kind of value which a study of the processes of thought may have for us, and the limitations of that value. Examination will lay bare the conditions under which thought reaches a conclusion which is justified by the event, so that when it fails to attain its goal we can put our finger on the point at which it began to wander from the straight path. In thought, as in other forms of skill, the principles of correct working, when they are discovered and plainly stated, help us to distinguish the right way from the wrong.

The study of thought has, then, both a theoretical and a practical value. Just as examination of the processes of nature leads to scientific knowledge, and as this knowledge can be applied in the making of various kinds of machines which, by combining the forces of nature, turn them to account, so examination of the processes of thought yields scientific—or exact—knowledge of the principles that underlie them when they are correct, and these principles can be applied both as rules to be followed in thinking and as tests to discover in what the error of a piece of unsound thinking consists. The system of knowledge thus attained is the

science of logic, which in its theoretical aspect is a systematisation of the method of knowledge.

As has been said, there are common principles in all thought which are the same whatever the matter with which it deals. Some writers have restricted logic to these formal considerations, and divorced it from any relation to the truth or falsity of the matter thought about, holding the sole test of its validity to be self-consistency.

The natural result was to bring logic into disrepute. For all real thought deals with some matter, and tries to gain knowledge of that matter. If logic is to be of interest to the ordinary mind it must consider how to reach true premises and how to test their pertinence and sufficiency, as well as how to reason correctly from them. A theory of thought should recognise that form and matter are always combined.

Though constant in the most general nature of its processes, our actual thinking differs in detail according to differences in the matter with which it deals. Each science is concerned with some one department or aspect of the world as it exists, and tries to conduct its researches in the way that will give the clearest understanding of that particular kind of relation and process.

In so far as the form is constant the matter may, indeed, be represented symbolically, just as in algebra the concrete terms of a problem may be replaced by letters. So the possible formal relations may be examined apart. But as the value of algebra is in solving actual problems, into the terms of which the symbols can be re-translated, so it is with logic.

Just as the interest of ethics, or the science of conduct, lies for most people in the help it gives in trying to live a good and happy life, so that of logic is commonly found in the help it gives in thinking correctly. Because of this practical bearing, each is called a *normative* science; that is, a science

that sets forth a norm, or standard of right. To regard either as only an art—that is, to see in its principles *merely* practical rules to be followed—is to reduce conduct in the one case, and thought in the other, to rule of thumb, which, as we have seen, is unfitted to adapt itself to change.

Logic, then, may be defined as *the science which treats of the principles of valid thought.*

(ii) Natural and Scientific Logic.—Logic helps those who study it in their attempts to think correctly just because it is a compendious statement of methods of thinking which experience has found to be trustworthy. The power to think is born in every one of us, and life compels us to exercise it to a greater or less extent. Experience teaches us how we have gone wrong in certain cases, and in that way we learn to avoid such errors in similar cases: a burnt child dreads the fire.

There is, thus, a kind of natural logic which life itself teaches all men. And in many this logic of life, or 'common sense' as it is called, attains a high degree of efficiency. Its power, however, is proportioned to the familiarity of the situations with which it has to grapple: the greater the novelty the less sure is the decision of the common sense of experience.

Scientific logic is not something different, but just this natural logic made more precise and more general, so that it becomes applicable to the widest range of topics. It lays bare the general principles which the man of shrewd common sense applies without knowing that he does so.

Scientific logic is the outcome of deliberate reflexion on thought; natural logic is the immediate application of the remembrances of past experiences to meet the present case. The former surpasses the latter both in its adaptability to new circumstances and in its ability to justify its conclusions by giving the reasons which lead to them.

But scientific logic cannot make bricks without straw: it

cannot invent truth, but only estimate and use evidence. It has, nevertheless, too often been assumed in the history of thought that if only logic be applied with skill it can extract certain truth from any material. This has led to much verbal disputation, and accounts for the slight esteem in which the educated man of sound common sense often holds the science.

It is true that the application of logic to any particular case shows that only one valid conclusion can be drawn. But that conclusion must often be that no certainty is attainable with the amount of knowledge at present available. One of the most valuable results of a clear grasp of the principles of logic is a refusal to pronounce with certainty on insufficient evidence, and a recognition that in many cases the best guide both to action and to thought available to us is a weighing of probabilities.

(iii) Dialectic.—The application of logic to formal argument is called Dialectic. It was the popularity of dialectic with the ancient Athenians that led Aristotle to investigate the principles involved in successful arguments, and to give the world the first attempt at a systematic treatment of the conditions of validity of thought.

The practice of dialectic was a recognised part of education during the middle ages, and it undoubtedly cultivated a power of keeping to the point and of seeing distinctions which is certainly not conspicuous among the men of to-day.

But the tendency of all disputation is to evoke the desire to establish the case one is arguing rather than to arrive at the truth; to entrap one's opponent and to hide one's own real defeat under some verbal quibble. In short, the practice of dialectic makes men skilful in the use of fallacies as well as in the use of pertinent argument.

The most profitable dialectic is that which each one uses with himself, arguing doubtful cases from both sides and bringing forward objections to the conclusion to which he is inclined, as well as considerations in its favour.

(iv) Use and Abuse of Logic.—The study of logic, then, should make clear to us the principles which can be violated only at the cost of error. It should put us on our guard against prejudices and mere acquiescence in the customary; for it insists on the necessity of evidence, and on the importance of weighing and testing the evidence available before drawing a conclusion. In short, it makes explicit in our minds the principles of clear thinking, and practice in forming and examining arguments under its guidance should form the habit of inquiry and of withholding judgement until we have sufficient pertinent evidence to justify a conclusion. People do not like being in doubt, and so often take sides in a dispute, or advocate an opinion, on no rational grounds. A logical training should at least make us aware that when we do either we are acting foolishly.

At the same time it must be remembered that logic cannot be the only ruler of life. Thinking is only part of our lives. And even in thinking there are always influences determining us of which we are at the most but dimly aware. Even the keenest psychological analysis will not make them all clear, and psychology does try to take the whole of life as its province and to determine how that life advances.

But logic is concerned only with a part of a part of life. Of such vital forces as love, sympathy, anger, ambition, and the like, it can take no account. Even in thought itself there is always present some interest or desire, and obscure factors which we cannot clearly set before us. Who can state fully so ordinary a thing as his knowledge of a friend, or even the evidence on which he recognises him in the street? Of none of this can logic take cognisance. It deals only with that explicit part of thought which we can express clearly in words.

This explains the notorious fact that controversy seldom convinces anybody. Indeed, in matters which concern us

personally the clearest logic affects us little unless it enlists on its side our imagination and sympathy.

But what logic can do is worth doing. Its practical study gives a tendency to our minds not to be led away by specious and showy rhetoric, and neither to be credulous dupes nor conceited sceptics, like the man who declared that he would believe nothing which he could not see, and who was somewhat dumfounded when asked whether he believed in the presence of brains in his own head.

CHAPTER II.

LANGUAGE AND KNOWLEDGE.

1. Functions of Language.

(i) COMMUNICATION.—Advance in knowledge implies co-operation. Were it otherwise, so that each man had to begin afresh the task of understanding the natural forces which act upon him and to pursue it unaided by his fellows, but a small minority would be likely to learn enough to avoid an early death, and it is very improbable that any would succeed in doing more than that. Progress is possible only because the experience of each can be made available for the guidance of his fellows, and because the results of the labours of one age can be passed on to ages that follow. Cultivation of the earth becomes more efficient, and rude tools develop into complex machinery, but each new contrivance simply improves upon one already in use, and can do so because a clearer insight into the needed co-ordination of natural forces has been attained. Even the greatest inventor adds little to the accumulated knowledge of the race in comparison with what he has received from it.

Co-operation, then, implies communication of thoughts and aims, successes and failures; and, as we all know, men communicate by language. Especially important to mankind at large is written or printed language, because it has so much wider a range both in space and in time than has the speech which it represents. Every increase in the facility of trans-

mitting language has marked an advance in civilisation. The invention of writing was the greatest step mankind ever took on the forward way; that of printing gave an enormous impetus to thought, of which the effect is continuously unfolding itself in extended knowledge; that of the application of electricity to telephone, telegraph, and wireless telegraphy, marks a more recent development, the results of which cannot be predicted. Taken together, these inventions go far to annihilate space and time as obstacles to the interchange of thought and knowledge between men.

(ii) Individual Meanings.—On the most cursory examination, language, whether spoken or written, is seen to be a system of signs which have no likeness to the things to which they refer. There is no similarity between either the appearance to the eye of the written word 'dog,' or the sound of the spoken word, and the animal which it indicates. It is simply because everybody refers the same word to the same thing or event that communication by language is possible. We understand the language of another just as far as our experience enables us to refer his words to things or events. When it fails to do so he is speaking what to us is really a foreign language.

But the fact that all people speaking the same language refer to the same things when they use the same words often leads to a false assumption. Because we use the same words in the same reference we are apt to believe that we all have the same ideas of the things spoken of; that is, that the words have precisely the same meaning for all who use them.

This is not so at all. I may talk to an old shepherd about his sheep, but to me 'sheep' means far less than it does to him. Similarly, the statement 'There was a Zeppelin raid on London last night' conveys one meaning to the man who quietly reads it in his morning paper in a place far removed from London; another to one who saw and heard the airship, and witnessed from a safe distance the explosion of the

bombs; and yet a third to one who narrowly escaped with his life, and that not without injury, from one of the houses destroyed. The intensity and variety of the personal experience in the last case make the words mean far more than they do in the second case, which, again, has a fullness and vividness of detail wanting in the first.

So it is throughout. What a word means to each one of us is what he has learnt in his experience about the matter to which it refers. A child picks up the names of things from the speech and actions of those around him, and things interest him mainly from the practical point of view. The use he can make of them gives them their value in his eyes. A peach is something to be eaten, not to be admired for its colour or its bloom; a knife is a thing that will cut other things.

With all of us this practical reference is the most important part of the meaning of the names of many things we use in daily life. Most people know little of trams and steam-boats beyond their use as means of travel, and that little is of quite minor importance to them.

Additional knowledge is united with this practical nucleus to the extent to which some interest leads us to investigate the object more fully. For example, we all first knew a watch as a producer of ticks, afterwards as a marker of time, and in relation to that we know that there is a system of wheels and springs which somehow causes the hands to move at a fixed rate. There the knowledge of most of us stops. But a boy who is apprenticed to the trade of watch-making learns much more than this. In a few cases his knowledge even becomes sufficient for him to invent some improvement in the construction of watches. Throughout all this developing knowledge of a watch, the word refers to the same things, but its meaning for the boy is steadily growing.

It is the same in every example we could take, and it thus becomes evident that sameness of words implies sameness of

reference to things, but not sameness of thoughts about them. The meanings which words have for different people correspond, but are not identical.

The fact that words mean to us just what experience has taught us about the things to which they refer is very obvious when we take as examples people instead of things, or events that involve our interests in place of those that do not. The same politician is a heaven-born statesman to his admiring followers and a rhetorical humbug to his opponents; the issue of a lawsuit appears in a very different light to the successful litigant from that in which his defeated opponent sees it; the treaty which ends a war does not seem of equal righteousness to each of the nations which sign it; the repartee which is simply witty to him who makes it is apt to be grossly rude in the eyes of him who suffers it; the caning is not the same thing to caner and to caned.

Nor does the same person or thing always have the same value for us; that is, mean the same to us. A friend who has treated us unkindly is for us a different person from what he used to be; an occupation loses its charm as years advance: the collecting of stamps or the spinning of tops no longer seems worth our while, the stirring stories of piracy which once enthralled us we now find dull and crude. Yet, looked at apart from our likings and dislikings, each of these things is what it was. It is our attitude towards them, not they themselves, that has changed.

Such examples bring out the fact that our thoughts are determined by our feelings and desires as well as by our knowledge. Those feelings and desires give a value to people and things which makes them what they are for us. Mathematics appears under a very different guise to one who loves the study and to one who detests it. But this personal attitude does not change the nature of mathematics as the science of the relations of space and quantity, and it is to that science that both students refer when they express their

attitude towards it. So in every case we see that the reference is the same, it is the thought that varies. And the thought is the meaning of the word for each person.

(iii) Common Reference.—It follows that the same words do not have exactly the same meaning to any two persons, for no individual has had exactly the same experiences as another. What is common to all, and so makes communication possible, is the correspondence in reference to the matters on which we speak.

This involves some agreement as to the kind of thing a word stands for—a meaning common to all who use the word intelligibly. This common meaning is only part of the actual meaning for any one of us, and the more our interests and feelings are involved in the matter the smaller a part it is. Nevertheless, the common meaning is the most important for the use of mankind. To very few persons does it matter whether you or I like or dislike mathematics, or whether we prefer pitch-and-toss or poetry. But mathematics plays an important part in the thought which reads the riddle of the universe, and poetry has revealed man's heart to himself. So, in the estimates we form of these things we really judge ourselves, even though our self-esteem may make us think it is the things we are approving or condemning.

(iv) Talking and Knowing.—We understand each other's language, then, because we use the same terms of reference to the things of the world. But we reach to the thoughts of another only to the degree to which our experience of the matter in hand has been similar to his. A child cannot enter into the jealousy of an Othello, the inner warfare of a Hamlet, or the sorrows of a Lear. All such intense emotions are beyond the range of his experience, and the words convey to him little or nothing of the states of soul Shakespeare is portraying for us. But in a sense he understands the language, for the words are familiar to him and have a meaning in so far as he can refer them

to something called by the same name in his own experience.

Here is a fruitful source of self-deception. We are all prone to think that power to talk about a subject implies real knowledge of it. But the talk may be little more than the echo of words read or heard, but never taken into the life by sympathy and imagination. Words can register for us only the experience with which they are connected, and that may have been little more than the hearing or reading the words themselves. Thinking is hard work, and men often evade it by repeating statements of the opinions of others. Increased power to talk of anything may show increased knowledge of it; but, on the other hand, it may show only an increased memory of what other people have said about it. "As men abound in copiousness of language; so they become more wise, or more mad than ordinary. . . . For words are wise men's counters, they do but reckon by them: but they are the money of fools, that value them by the authority of an Aristotle, a Cicero, or a Thomas, or any other Doctor whatsoever, if but a man."[1]

2. **Knowledge from Testimony.**

Knowledge of words, then, is not necessarily knowledge of things. Acceptance of the statements of others may even hinder increase of real knowledge, and that, not only because we may be content with a mere remembrance of words which have little meaning for us, but because trust in the source of the information may lead us to reject any evidence which throws doubt on it. "Galen thought that the arteries carry the vital spirit from the heart to all parts of the body; and if this is so, there must be a hole in the septum of the heart to allow the spirit to pass from the arteries of the lungs into the arteries of the rest of the body. He taught, therefore, that there is such a hole, and for fourteen hundred years

[1] Hobbes: *Leviathan*, pt. i., ch. 4.

anatomists believed him, and in spite of the plain evidence of their senses, followed his teaching, and believed that a hole is there, although they could not find it." [1]

Yet, unless men accepted the testimony to fact of their fellows, the co-operation on which the advance of knowledge depends would be impossible. Indeed, no one could live in a community and disbelieve all that is told him, though few are aware, until they inquire into the matter, how much of their knowledge is thus acquired at second-hand. On the other hand, no adults are so credulous as to believe all they hear or all they read. It is important, then, to have an idea of the kinds of tests we may reasonably apply to the statements of others about facts outside our own experience.

When an assertion is made to us we may either accept it as true, reject it as false, or keep an open mind about it. In the first case we regard the evidence offered as sufficient, and we believe; in the second we treat the evidence as of no worth, and disbelieve; in the third we do not reject the evidence, but consider it insufficient. In current speech both the first and the last of these states are called belief. When we say we believe something we often do not mean that we are sure of it, but that we are in doubt about it, as when a servant informs a visitor that she believes her mistress is at home but will go and see. In the stricter sense 'I believe so-and-so' means 'I am assured of its truth.' Disbelief is equally a state of certainty; it is belief that the statement is false. The state of mind opposed to each is doubt.

If the matter be one of indifference to us we may be content to have no opinion on it one way or the other for an indefinite time; but if it be one that touches our interests we find a state of doubt uncomfortable. Then we often make up our minds according to our desires or fears, and

[1] Dr. C. Mercier: *Causation*, p. 203.

say we believe when really we only like and hope, and disbelieve when in truth we merely dislike and dread. A logical training should keep us from basing our beliefs on a confusion between the facts of the world and our feelings towards them, and should help us to form the habit of solving doubt only by the accumulation of evidence.

3. Tests of Testimony.

(i) Agreement with Knowledge.—Now, an assertion is either credible or incredible to us independently of the source whence it comes. When we examine the ground of this we find it in the relation of the alleged fact to our existing knowledge and mode of thought.

Here we have to be on our guard against taking prejudice for knowledge. We all hold opinions simply because we have lived among people who have held them. So we have accepted them as a matter of course; they have grown with our growth and strengthened with our strength, and often they colour our whole mental lives. But, as we have never examined the evidence for them, they are merely prejudices to us, whether in themselves they be true or false.

If we read such prejudices into a statement offered for our acceptance we really change it, and accept or reject something different from what it is meant to convey. If, on the other hand, we accept or reject it simply because it agrees or conflicts with a prejudice, we are acting irrationally.

If, however, it is not prejudice but real knowledge which is opposed to the statement, we do well to reject it; for we cannot knowingly admit inconsistency into our thoughts. But if it is only outside our knowledge, and not repugnant to it, we ought to remain in doubt about it. If it accords with our knowledge we should be prepared to accept it, provided the evidence is satisfactory.

Of course, hundreds of instances of all these modes of dealing with statements are found daily in the lives of all of

us. We accept or reject or doubt almost automatically in the vast majority of cases of a customary kind and of little importance. It is only when a statement challenges our attention by its novelty or by its obvious bearing on our interests that we meet it with deliberate criticism. And then we often have to conclude that we cannot be certain, and that we must act on the highest probability.

Agreement with accumulated knowledge and accepted systems of thought is, then, the first test. As knowledge increases the sieve becomes finer.

In the fourteenth century the men of Europe knew little of distant countries, and they did not conceive of nature as a system of constant forces; so, many things were credible to them which men of to-day unhesitatingly reject. Sir John Mandeville could assert that diamonds "grow together, male and female, and are nourished by the dew of heaven; and they engender commonly and bring forth small children, that multiply and grow all the year. I have oftentimes tried the experiment, that if a man keep them with a little of the rock, and wet them with May-dew often, they shall grow every year, and the small will grow great."[1]

We do not hesitate for a moment in rejecting this testimony, because it conflicts with our knowledge of minerals in general and of diamonds in particular. But to the great traveller's contemporaries diamonds were almost unknown, and the unknown might well be the marvellous.

Nor have we faith in the same narrator's "snails so great that many persons may lodge in their shells, as men would do in a little house."[2] For though such snails would not be in contradiction to any known law of nature, yet the size of all known snails makes their existence highly improbable, and it is certain that had they existed they would have been found by travellers in the East Indies, where the story places them.

[1] *Voyages and Travels*, ch. 14. [2] *Ibid.*, ch. 18.

In applying the test of congruence with knowledge we should bear in mind that personally each one of us is master of but a very small part of the total knowledge of mankind, and so should hesitate in rejecting assertions simply because they are outside the range of our personal knowledge. In such cases we should remain in doubt until the evidence of experts in that particular branch of knowledge decides the matter for us.

(ii) Trustworthiness of Witnesses.—As testimony is the communication of the experience of another, the question of the trustworthiness of the witness is raised. Mandeville asserts that he had "oftentimes tried the experiment" and found diamonds to increase in size. That experiment any-one who possesses a diamond and "a little of the rock" on which "men commonly find them in the sea" may repeat. But he will not expect any growth in the size of the diamond. He knows, indeed, that diamonds are not found upon rocks in the sea, and if sea-salt be meant, he would attribute any apparent increase to precipitation of particles of salt when the "May-dew" is evaporated by the heat of summer. So, on the face of it, we have the alternative of supposing either that Mandeville was deliberately preferring his love for the marvellous to adherence to the truth, or that he was a most incompetent observer. In either case he is an un-trustworthy witness.

So it is generally. Good faith is not a guarantee of com-petence. To ensure that, we need in addition to be sure that the witness was in a position to know the facts, that he is a competent observer, and one who can interpret correctly what he observes.

It is sufficiently difficult to apply these tests satisfactorily when we know who is the witness, but in many cases we do not know even that. This is so with much of the con-temporary evidence on which history mainly rests. Even when we know who wrote the record we cannot be sure that

he is not simply repeating what another has told him, or even mere common gossip.

It is the aim of modern historical criticism to determine the authenticity of records of the past, and to trace to their sources the statements they contain. The results are at times startling. For example, "*The Travels of Sir John Mandeville* had been a household word in eleven languages and for five centuries before it was ascertained that Sir John never lived, that his travels never took place, and that his personal experiences, long the test of others' veracity, were compiled out of every possible authority, going back to Pliny, if not further."[1] So the question whether the story of the growth of diamonds, proved by personal experiment, arose from excessive power of imagination or from defective power of observation is settled. For it no longer involves the good faith of a known witness, who in general estimation was honest in personal statements, though sharing the credulity of his age, but of an unknown recounter of marvels, living at an unknown time, and of whose veracity and competence we know nothing. It is reasonable, therefore, to adopt the more probable supposition that the whole story of the "experiment" is one of those 'touches of reality' which with the uncritical add much to the speciousness of an account which in itself might well be judged open to suspicion.

(iii) Common Report.—As an illustration of how rapidly and seriously popular report distorts historical facts the following may be cited. Schaschek, the chronicler who accompanied Lev, 'the Bohemian Ulysses,' on his journey through Western Europe in the fifteenth century, records that at Blaye they learned that "This city was held by the Kings of England for one hundred and fifty years. But it was won back by a certain prophetical woman, who, indeed,

[1] Miss A. D. Greenwood in *Cambridge History of English Literature*, vol. ii., pp. 78-79.

recovered the whole kingdom of France from the English. That woman, although born of a herdsman, was so ornamented by God with virtues, that to what matter soever she addressed herself, it was brought to a right end. Yet in her last battle being captured by the King of England and taken to England, and having been there by his orders placed upon a brazen horse and led throughout the city of London, she was at length, by the violence of flames, done to death and transmuted to ashes, which were afterwards scattered abroad in the sea."[1] This was the embroidered version current only thirty-five years after the event! The central facts of the humble origin, success, and cruel fate, of Joan of Arc are true: the attendant circumstances are fictitious.

This is typical. Falsification of detail, through imagination, love of the marvellous, desire to astonish and impress, lapse of memory, or mere seeking for rhetorical effect, may be confidently looked for in all popular versions of fact: the difficulty is to know how much is picturesque elaboration and how much is simple truth. Generally it is safer to accept a statement that an event *has* happened than to credit details of descriptions of *how* it happened.

Not only in records of the past is popular report to be cross-examined. As Dr. C. Mercier says: "'They say' is an authority that is accepted with unquestioning submission, without even a query as to who are the 'They' who say it." Among other examples he gives this: "There is a prevalent belief, for instance, that cigarette-smoking is more injurious to the smoker than the smoking of pipes; and this belief is widely and firmly held on no better ground than the belief that it is unlucky to look at the new moon through glass. Occasionally we may obtain the assurance that 'doctors have said it,' but it is usually found that 'doctors' is but another

[1] Quoted by Mrs. H. Cust in *Gentlemen Errant*, p. 61.

expression equivalent to 'They.' . . . It is clear to anyone who gives a moment's thought to the matter, that to determine whether cigarette-smoking is or is not more deleterious to health than pipe-smoking would require a very long and laborious course of experimentation, such as no one has ever yet undertaken, or an accumulation of non-experimental evidence, such as has certainly never been attained."[1]

(iv) Evidence of Independent Witnesses.—It may be objected that common report is really a case of corroboration by a number of witnesses. This cannot be granted. We have no witnesses before us: we cannot trace out the original 'They,' and do not even know whether 'They' were, in fact, more than one. Corroboration of testimony by *independent* witnesses is valuable, and may justify even certainty. It is, for instance, one of the chief means of critical decision as to the authenticity of various readings in copies of the same old manuscript. But mere number of witnesses is of no avail if there is internal evidence that they all originated from a common source, as, for example, that they all make the same mistakes. This one consideration takes away the strength of numbers from common report. In no case can that legitimately do more than suggest a line of inquiry.

Yet, the uncontradicted evidence of many witnesses, supported by all kinds of corroborative facts, is the surest guarantee we can have of matters that do not come within our own experience. He would be a sceptic indeed, and we should strongly suspect his sanity, who should refuse to believe in the existence of Germany, on the ground that he has only hearsay evidence.

Even when the witnesses are few, we reasonably receive their testimony if we are convinced of their competence and good faith. We should not, for instance, doubt the an-

[1] *Causation*, pp. 200, 201.

nouncement of an astronomer that he had discovered an additional moon in the system of Jupiter. We know that if he were in error he would very soon be corrected by other astronomers, though we ourselves cannot test the statement.

4. Fallacious Use of Language.

Our inquiry has shown us that the meanings of words are not fixed and unalterable, like little pieces of stone to be fitted into a mosaic pattern. Language is a living activity, and is always used in certain definite circumstances, for a certain purpose, and with a certain reference to the hearer or reader. All these together determine what words shall be used, and in what order they shall occur. Thought spontaneously clothes itself in language, and it is only when we feel an exceptional need to express ourselves with clearness and precision or with elegance, that we criticise this outpouring of our thoughts in words.

Nevertheless, it is possible to keep a kind of critical eye on our speech, and still easier to examine our writings, with the definite intention of forming the habit of a skilful use of language, so that the right forms of words to express our exact meanings may readily present themselves. We English people take less pains with this than do our French neighbours, and, as a result, slipshod speech and writing, which have to be interpreted by hearer or reader, are more common with us than with them. And confusion of speech is an almost constant attendant on confusion of thought. From this combination arises much erroneous and inconclusive reasoning.

We will briefly examine the pitfalls which faulty language most commonly digs in the path of thought, and, for convenience of reference, we will class them under the names which they have borne among logicians for many centuries.

(i) Ambiguities of Words [Aequivocatio or Homonymia].—The question to be decided is not clear: the

thinker is vague as to what his words really imply, so he wanders in thought from one shade of meaning to another and assumes to be true of the one what is only true of the other. Of course, if the matter is being discussed between two disputants the confusion is likely to be still greater. Each is largely fighting in the air, for each interprets his opponent's words in a different sense from that in which they are intended to be used, and does not feel called upon to make allowances for any unavowed shifting of ground in which that opponent may indulge.

A fallacious use of language thus resolves itself into confusion of thought helped by ambiguity of terms. Words which refer to things or relations conceived by the mind, but not observable by the senses, are specially liable to be thought vaguely and obscurely. We have already noted stricter and looser meanings in which 'think' and 'believe' are used. Evidently, unless one is on one's guard one may slide from one to the other use, and deceive both oneself and others.

The following anecdote is in point: "The intelligent child was listening intently at the breakfast table while her elders were discussing the spread of the cocaine habit. It was only when the conjecture was hazarded that the evil was as prevalent in the provinces as in London that she intervened in the conversation. 'Oh, no, mother,' she said, 'I don't think so, because Miss Smith was only telling us the other day that London is called the land of Cockaigne.'"

Every student of history knows how difficult it is to get a correct insight into how people of an earlier time lived, and what they thought, valued, and desired. Some of this difficulty is due to changes in the meanings of words. The purchasing power of a penny, or of a pound, is very different now from what it was five hundred years ago, yet the same names are used. It is fatally easy to draw all sorts of unjustified conclusions as to the wages of the working

classes, the cost of houses or of ships, and, indeed, as to everything into which purchase enters. Similarly, the absolute government of the Roman Emperors, or that of King John or Henry the Eighth, bears little resemblance to the English parliamentary government of our own day, and inferences drawn from identity of name would be pretty sure to be fallacious.

A cognate error to that of applying the same name to different things is the use of different words to represent the same thing. In social and political disputes the same measures are estimated very differently by men of opposing views. What is the removal of injustice to the one side is an invasion of sacred rights to the other. Many a proposal for the readjustment of taxation, advocated by its supporters on the ground of a just adaptation of burden to capacity has been even refused a hearing by many because it has been labelled 'confiscation' by its opponents.

In party politics it seems to be especially difficult to credit the other side with either honesty or intelligence. Though written more than a hundred years ago in reference to the government of Ireland, the following passage is not wanting in pertinence to our own day: "But what do men call vigour? To let loose hussars and to bring up artillery, to govern with lighted matches, and to cut, and push, and prime; I call this not vigour, but the *sloth of cruelty and ignorance*. The vigour I love consists in finding out wherein subjects are aggrieved, in relieving them, in studying the temper and genius of a people, in consulting their prejudices, in selecting proper persons to lead and manage them, in the laborious, watchful, and difficult task of increasing public happiness by allaying each particular discontent. . . . But this, in the eyes of Mr. Perceval, is imbecility and meanness."[1] Where the same word implies such different courses

[1] Sydney Smith: *Peter Plymley's Letters*, Let. 9.

of action, and the same course of action is designated by such opposed terms, it is evident that the arguments on the one side are little likely to appear convincing to the other. The history of a century has proved that this has, indeed, been the case.

(ii) Confusion between Meaning and Origin of Words [Figura Dictionis].—Words of similar derivation are easily assumed to be of similar meaning. This is not always the case. For example, though 'joyful' means full of joy, and 'distrustful' full of distrust, yet 'pitiful' does not mean full of pity, nor 'faithful' full of faith. One may 'presume' without being presumptuous, and 'conceive' a theory without being open to the charge of conceit. 'Strong' language has no necessary connexion with strength of either will or action, though such a connexion is often assumed.

Much false argument has had no other foundation than unconscious alternation between the accepted meaning of 'desirable' and a supposed meaning analogous to that of 'visible' or 'audible.' To say that anything is 'desirable' implies that it *ought* to be desired, but a cat on the tiles may be audible in the night without imposing on me the duty of listening to it. A Zeppelin is both visible and audible; as *The Daily Telegraph* recently informed us: "When first we heard the Zeppelin the engines were distinctly audible." But we do not feel bound to wish for a visit from one every night, so that we may see and hear it, though the Germans may so desire. Arguments based on similarity in the forms of words are, therefore, apt to lead us astray.

(iii) Confusion between Distributive and Collective Use of Words [Compositio and Divisio].—What is true of a whole need not be true of its parts. But when the same word can be used for each it is not difficult to pass unconsciously from the one application to the other. The spendthrift may ruin himself by assuming in practice that if he can afford a choice between a variety of expenses he can

afford them all: that is *Compositio*. Or, by *Divisio*, a stingy person may excuse his meanness in incurring none of the expenses on the ground that he cannot afford them all That even Cabinet Ministers are not immune from this fallacy is seen in the following extract from a speech in support of compulsory military service: "Compulsion and voluntaryism are not inconsistent in a democratic nation. Compulsion simply means the will of the majority of the people—the voluntary decision of the majority. . . . So compulsion is simply organised voluntary effort." [1]

Ambiguity is often due to the possibility of using such words as 'all,' 'some,' 'no,' either collectively or distributively. In 'All the arguments of my opponents failed to convince me' the reference is to the whole mass of reasoning brought to bear upon me. But if I say 'All the arguments were weak and childish' I express an opinion on each of them individually. It may be true that 'No poet wrote this doggerel,' and equally true that 'Jones is no poet'; but we cannot thence conclude that he is the author of the condemned verses, for 'no' is used distributively in the first, and collectively in the second premise. A pleasing example of this confusion was recently supplied by the correspondent at Salonika of *The Daily Graphic*, who assured us that "Every seat in every café . . . is occupied by thousands of men in uniform."

(iv) Ambiguities of Construction [Amphibolia].—English is an analytic language, in which the sense depends largely on construction. This offers abundant opportunities for ambiguity, which are by no means neglected either by speakers or by writers. Indeed, we all have to form the habit of largely disregarding the literal meaning of what people say, and attributing to them a different meaning. Usually this is easy enough, but at times we are led into

[1] Speech of Mr. Lloyd George, *The Daily Chronicle*, May 15th, 1916.

positive error, and at others we are left wondering. What is the wish expressed in: "I hope that you the enemy may slay"? In the following passage does the phrase "any more than" connect the words that follow it with "state" or with "religion"? "Locke tells his antagonist that it does not follow that the state is bound to protect religion any more than the East India Company."[1]

As De Morgan remarks: "Every one should be aware that there is much false inference arising out of badness of style, which is just as injurious to the habits of the untrained reader as if the errors were mistakes of logic in the mind of the writer."[2]

Examples can be found almost at pleasure, and the hunt for them is not only amusing, but helpful in the formation of the habit of unambiguous construction. When Sydney Smith asks "Can anything be more distressing than to see a venerable man pouring forth sublime truths in tattered breeches?"[3] the intended pathos is lost in the unintended humour.

A daily paper recently told us that "Later a Zeppelin was picked up by searchlights flying at a great altitude." Of course, not a single reader believed what was *said*; every one implicitly re-arranged the sentence.

The proprietors who advertised in *The Times* "Service Flats: Provide catering and attendance equal to that obtainable at the best hotels at half the cost," would certainly object to be taken at their word.

Recently a Northamptonshire teacher received the following letter from the mother of an absent scholar: "onored sir, yesterdy a boy threw a stone in my Tom's eye, and he

[1] Sir Leslie Stephen: *English Thought in the Eighteenth Century*, vol. ii., p. 150.

[2] *First Notions of Logic*, p. 24.

[3] *Peter Plymley's Letters*, Let. 9.

can't see out of it will you please see into it." Such ambiguity in the use of 'it' is not uncommon.

Great as may have been the bulk of Henry VIII., the implication in the following advertisement rather staggers one: "Brittany.—To be sold, a Park of 200 hectares, enclosed by walls. Copses. Trout fishing. Fields well watered. Castle date Middle Ages, with largest donjon in France, having served as Residence to Henry VIII. of England. For particulars address Me. Morice, St. Aubin d'Aubigne (Ille et Vilaine)."

Faulty punctuation is a frequent cause of ambiguity. When a daily journal informed us that "Imports in truth have been so small that the run on home produce has been more or less forced" we were at first inclined to think it an excuse for the newspapers, but probably the reference was meant to be a wider one, which would be made apparent by the insertion of commas after "imports" and after "truth."

When a writer in *London Opinion* announces "I am sitting down with my pen in my hand filled with a cold resolution to lose my temper thoroughly," his meaning can be reached only by placing a comma after "hand."

The announcement that "The Gardens and Deer Park will be thrown open to the public. . . . Children under fourteen unaccompanied by their Parents and Dogs not admitted" is surely better in intention than in form. The insertion of a couple of commas would at least make it unambiguous.

Nor can we accept as it stands this statement of a provincial journal: "A ship's apprentice who attempted the rescue of a man in shark-infested waters to-day, at Newcastle, received the Shipping Federation's diploma and medal." But take out the commas, and put one in after "waters," then we have a credible record.

(v) Ambiguity of Emphasis [Accentus].—Change of emphasis may suggest a serious change of meaning. For instance, when two persons are in question emphasis on a

personal pronoun will often imply a contrast which is not implied if there is no such stress. If a candidate for election when comparing his policy with that announced by his opponent were to remark "But *I* do not promise what I cannot perform," he would be understood to imply that his adversary was not scrupulous in that respect.

Doubtless Mark Antony's repeated assurance that "Brutus is an honourable man" was given in a tone that suggested to his hearers the advisability of inserting a negative. So with all irony and sarcasm. But there are prosaic souls that take everything in the most literal sense. These are always misled by irony, and often by simpler forms of humour. We can but pity them. For their disease no remedy is known.

CHAPTER III.

JUDGEMENTS AND PROPOSITIONS.

1. Nature of Judgement.

(i) JUDGEMENT IS THE SIMPLEST ACT OF THOUGHT.—The aim of all our thought is to explain our experience, either of things and events which are independent of our feelings, desires, and wishes, or of the way in which such things and events affect our lives. Thus, the very simplest form of thought asserts something as true. We may think of a single word, and if we know to what it can be applied it has some meaning for us. Yet it does not represent a thought, but only a challenge to thought. If I think of 'book' or 'fire' or 'air-ship' I find myself making a variety of assertions about it; affirming that it is this or that, and denying that it is the other.

Similarly, when I hear a single word, it puts my mind into a state of expectancy. I listen for what is to follow to show me why it is used. Or I supply an assertion myself. If I should hear a man cry 'Fire!' or 'Thieves!' I should not suppose him to be simply announcing that the idea of fire or thieves had just entered his mind, alone and in no relation to anything else; I should assume that he was affirming the existence of fire or thieves in some particular place and in some relation to himself.

The most elementary piece of explicit thought is, then, an assertion or judgement, and, however many words it may

take to express, each judgement is one simple and indivisible act of thought. Indeed, we know that whether a single judgement be expressed by one word or by several is a matter in which languages differ. The judgement which the Latins expressed by the single word 'scribo' requires in English the two words 'I write'; the one word 'scripsi' is represented by the three words 'I have written.'

(ii) Words imperfectly represent Experience.—Words often mislead us, however, by their separateness. They induce us to think both of things and of our ideas of things as being as self-contained and independent as they are themselves. This is not the case. The very structure of language compels us to say 'Fire burns,' 'Magnets attract iron,' and we are led to think that fire can exist apart from the power to burn, and magnets without that of attraction of iron, in the same way as the words can be set apart without changing their form. That is quite a wrong view of the nature of things. Nothing would be fire which did not burn, nor would that which did not attract iron be a magnet, however great might be the resemblance in other respects. A bunch of artificial grapes may look very real to the eye, but we decide against their claim to be what they simulate immediately we test them by taste.

We can only describe an experience by enumerating its elements one by one, but this separation is only in our own thought. The experience itself is given us whole and undivided. We say, for instance, 'The leaves are being blown from that chestnut tree by the wind,' but the twelve words describe what we see as one single fact.

It is plain, then, that language can give only an approximate expression of our experiences, and if we suppose it to express them fully, and exactly as they occur, we are misled. Indeed, in thinking we do not usually put our thoughts into complete sentences at all; though we use language, often only what may be called 'key-words' are explicitly present in our

minds. Our thought can proceed much more rapidly than we can utter words, even to ourselves. But each key-word represents a step of thought; that is, a judgement.

(iii) Judgements express Belief.—As words, then, are the elements of which judgements fully expressed in language consist, so judgements themselves are the elements of continuous discourse. Even apparently isolated judgements have a connexion with the course of our experience. They do not grow out of nothing, nor are they made without a motive. Each of them brings to light a little piece of that life of feeling, desire, and thought, which we call ourselves.

From this nature of judgement as the revelation of a part of our mental life it follows that every judgement is the expression of a belief. To judge is to declare true. Certainly men can lie; but when they do so they do not express their experience, their words do not represent the judgement they have formed. Or, again, without intending to tell a falsehood, men may be mistaken and declare a falsity. Here their words do express their judgement, for they state their belief. It is the belief that is mistaken.

In every case the verbal expression of an actual or possible judgement is called a *Proposition*. It is plain, then, that though on its face a proposition makes a claim to be accepted as true, yet it may be false; and that, either because he who enunciates it may intend to deceive others, or because he is himself deceived.

(iv) Questions, Requests, and Commands, are not Judgements.—Judgements are the separate steps by which our thought proceeds on the way of knowledge. But each advance is attempted because some problem has come in our way and raised an obstacle in our path. Till we have solved it, our attitude towards it is not one of assertion but of interrogation. A question is not a part of inference, but a challenge to inference. Logic deals with the thought by which that challenge is met, but not with the question which

makes it. For no general principles can be discovered as to why or when various questions are raised in our minds.

Further, we not only think, but we wish and desire. We may express a wish in a judgement, as 'I wish so-and-so,' when it is an assertion about ourselves, and may enter into inference like any other judgement. But our wishes often relate to the actions of other people, and to secure their accomplishment we express them as requests or as commands. Here again we have no assertion; indeed, the request or command may be disregarded. We cannot, therefore, treat either as a step in inference. Consequently, such expressions of our feelings do not fall under the consideration of logic.

2. Forms of Judgement.

(i) CATEGORICAL, HYPOTHETICAL, AND DISJUNCTIVE, JUDGEMENTS.—Let us consider such a line of thought as the following: 'I have a headache. But if I try to work when I have a headache I shall accomplish little of any worth. Yet I must either do this work to-day or be put to grave inconvenience to-morrow.'

It will be seen that the argument has three steps, and that each step consists of a different form of judgement. The first, 'I have a headache,' is a bare statement of fact. It expresses an immediate experience, and its whole justification is in that experience. That is called a *Categorical Judgement*, which means simply a judgement of fact.

The second, however, states no fact. It does not assert that I have a headache, for I might make it with equal truth when I am not so afflicted. Nor does it assert that my work will be of little worth, for there may be times when that proposition would be far from expressing my estimate of its value. What it does assert is a relation, which experience has taught me to expect, between a condition and its results. It says that *if* work be done under the condition named it will be of little value. And it asserts this, not merely of to-

day, but as a general or universal truth; that is, it asserts that the relation will hold true in every case of the kind. It is, therefore, called a *Hypothetical* or *Conditional Judgement.*

In the third case the judgement again asserts no fact, but only a choice of alternative facts. Its justification is to be found in a system of related facts which is such that to-morrow's comfort depends on that particular work being accomplished to-day: it may be that the work is the preparation of a lecture which has to be given to-morrow. This is called a *Disjunctive Judgement*, because it disjoins explicitly the alternatives which the system makes possible.

All judgements are of one of these three classes. The first expresses fact; the second states a law, or uniform relation; the third unfolds a system. The first is the earliest immediate expression of experience, but each of the two latter evidently expresses the result of previous experience. Only when things and events have been examined and compared can we state that one will always result from another, or that certain alternatives are open before us.

(ii) DEVELOPMENT OF JUDGEMENT.—Though thus different in the way in which they express our knowledge, yet, as there is no break in our mental life, so there is no discontinuity between our modes of judging. We begin by judgements of fact, but we also begin with a natural tendency to generalise them; that is, to extend them to all similar cases. The child who has played with a dog is likely to call the first sheep he sees a 'bow-wow.' He classes it with the most similar familiar object. He is told 'No; it is not a bow-wow, it is a baa-baa.' So he learns to note as important, differences which at first he had not regarded. And generally, the use of names by those around us in childhood first classified for us the objects of which we had some form of experience.

Such classification keeps generalisation broadly within the bounds set as legitimate by the knowledge of the community. But within those limits it proceeds merrily. The child repeats

the striking of his spoon on his plate in full expectation that the delightful sound will be heard each time; he turns from the powder, unless it be well hidden by the jam, for he anticipates that he won't like it any more this time than he did last time. He does not put his generalisation into words, even to himself: he practises it rather than thinks it. But, as thought becomes clearer, and language more available, he is ready to meet a challenge of his refusal to take the powder by asserting that it is 'nasty.'

As life advances some of these implicit generalisations are corrected by experience; others are confirmed. So arises the distinction between what always is so-and-so, and what is so-and-so only sometimes. The thorns on a rose-tree always scratch, but the flowers are not always scented. Put explicitly in words these experiences become 'All thorns scratch'; 'Some roses are scented.' And of any particular rose-tree seen at a distance, the judgements made are equivalent to the propositions 'Its thorns will scratch'; 'Its flowers may smell sweetly.' If the bush is first seen at too great a distance to be identified with certainty, the natural expressions of these expectations are 'If that is a rose-bush its thorns will scratch'; 'If that is a rose-bush its flowers may smell sweetly.'

This last example shows that we may use the hypothetical form even when we cannot assert explicitly a universal relation. It is not that such a relation is not present, but that we cannot state with sufficient definiteness the terms between which it holds. The scent of a rose is not a mere chance, it is invariably found in some varieties, and is as invariably absent from others.

(iii) Modality of Judgements.—So far the judgements considered express facts and the observed relations of facts. But facts challenge us to explain them, though we do not always accept the challenge, and when we do are not always successful in the attempt. In this case little further examination is needed to show us that the reason why thorns scratch

is found in that combination of hard material and sharp point which makes the thorn a thorn. So now we can go further, and assert not only that thorns *do* scratch, but that, being what they are, they *must* scratch. This is called a *Generic Judgement*; that is, a judgement based on the nature of the thing of which it is made. It is, then, a judgement of necessary relation, and may be equally well stated hypothetically: 'If any thing is a thorn it will scratch.'

We have, then, three degrees of assurance in our judgements: a thing *is* so-and-so, or *may be* so-and-so, or *must be* so-and-so. This is called their *Modality*: the first is *Assertory*, the second *Problematic*, the third *Necessary*.

(iv) Necessary and Empirical Judgements.—When we can make such a generic judgement as 'Thorns, as such, scratch' we evidently imply that this is true of every thorn. We may express this more direct reference to experience by saying 'All thorns scratch.'

But we do not limit our assertory judgements to cases in which a necessary judgement underlies them. We learn from experience that a certain variety of rose—say the Hugh Dickson—is always sweet-scented. We may not know *why* this is so, and yet we feel no hesitation in generalising from the few specimens we have examined, and saying 'All Hugh Dickson roses smell sweetly.'

Here, as elsewhere, we are guided in the extent of our generalisation by the name, which, on the authority of others, we accept and attach to red roses of a certain tint and form and odour. In other words, were the scent wanting in a particular flower which looked like a Hugh Dickson, we should not revise our judgement and make it 'Some Hugh Dickson roses are scented,' but we should deny that the rose before us was a Hugh Dickson at all.

(v) Relation of Hypothetical and Disjunctive Judgements.—Were our knowledge of the varieties of roses adequate we could enumerate them all in a disjunctive judge-

ment, and say 'Every rose is either . . . or . . . or . . .' for as many terms as there are such varieties. But, in the first place, the number of varieties is continually being increased by rose-growers, and in the second place, our knowledge of existing varieties is likely to stop far short of the reality. So the disjunctive judgement will only be complete if we give as a last alternative some such indefinite term as 'some other variety I do not know.' Generally, however, when we use a disjunctive proposition we definitely intend to limit the alternatives to those we name.

What, then, is the relation between them? This question leads us to consider the connexion between the disjunctive and the hypothetical judgements. In the case of the roses, the alternatives are incompatible; so that, if a particular flower belongs to one variety it cannot belong to another. But alternatives are often proposed which only partially, or probably, exclude each other. I may not do the work to-day, yet the inconvenience feared for to-morrow may not be inevitable: I may be able to give the lecture with such preparation as may be crowded into a few stolen minutes. 'You are either deaf or inattentive' does not exclude the possibility of both weaknesses being present. So, though if we deny one of two alternatives we must affirm the other: 'If you are not deaf, you *are* inattentive'; yet from the affirmation of the one we cannot assume the falsity of the other, for, 'If you are deaf, you may—or may not—be inattentive.' The disjunctive form by itself does not guarantee that the alternatives cannot be true together.

The disjunctive judgement is also seen to be a development of the knowledge implied in a categorical judgement. It marks an analysis of the scheme of things to which the latter refers, and supplies a choice of more definite assertions about it.

(vi) NEGATION.—In the last example we were led to make a *Negative Judgement*, that is, a judgement which denies,

and we have seen that we may deny that a particular rose is a General McArthur because we assert that it *is* a Hugh Dickson. We must have a positive ground for our denial, or it would be a mere set of words without meaning. If I deny that a book is interesting it is because I find it unattractive; if I say a man is not honest it should be on the ground that I know him to have been guilty of fraudulent acts. There is no such thing in thought as a bare denial. The form in which we *express* our judgement may be simply 'That is not so-and-so,' but that is only the salient feature of the judgement, which is always in real thought 'That is not so-and-so because it is something incompatible with so-and-so.' Nor is denial made at random: it is always in reference to an affirmation made or suggested.

(vii) The Universe of Discourse.—This leads us to see that our thoughts at any one time are always within a certain range. When I say that a rose is not red, it is because to me it seems to be of another colour; it may be pink; or, if I am speaking from memory and the flower is not before me, I may believe that variety to be white or yellow. But the denial of red is the implicit assertion of some other colour.

Doubtless, it is possible to put together words into sentences which are grammatically correct but which have no meaning, and, therefore, are not propositions, as they represent neither actual nor possible judgements. We can say or write 'Courage is not red,' 'Cabbages are not very affectionate,' 'Every boy is equal to two right angles.' If we seriously did so our sanity would be justly open to suspicion; for no sane person could make such *judgements*, and a sentence claims on the face of it to express a judgement. As logic is an examination of thought it has nothing to do with such exercises of perverted ingenuity: it simply ignores them.

All propositions, then, are to be understood as limited in

their application within the field to which their terms indicate that they belong. This field is called the *Universe of Discourse,* because within its bounds judgements about 'All' and 'None' are universally true.

(viii) Summary of Forms.—Summing up these results we may say—

(1) That judgements are either *Affirmative* or *Negative.*

(2) That categorical judgements may be—

(*a*) *Universal: i.e.* make the judgement explicitly of all the cases under consideration. 'All these things are so-and-so.' This may be intended either distributively or collectively. In the latter case the negative universal is formed by the simple insertion of 'not'; 'All these things (together) are not so-and-so.' But in the former case, when the 'All' of the affirmative may be equally well written 'Every,' that form is misleading. 'All these things (separately) are not so-and-so' would be true if the assertion that they were so-and-so were false of any one of them. 'Not all' (separately) simply means 'some,' and the negative judgement really stated is particular. The distributive universal negative should, therefore, be written 'None of these things are so-and-so.'

(*b*) *Particular: i.e.* make the judgement of an indefinite number of those cases; certainly of one, possibly of all; our knowledge is not sufficiently complete for us to say. So we prefix 'Some' to the name, and say 'Some of these things are so-and-so' (affirmative), and 'Some of these things are not so-and-so' (negative). We must bear in mind that 'Some' does not mean 'Some only' but is quite indefinite, excluding nothing but 'None.' So it does not shut out the possibility that, as a matter of fact, it may in some cases be 'All.' It means simply that our knowledge is limited as to the range within which the affirmation or denial is true.

(3) That a similar distinction can be made, on similar grounds, between universal and particular hypothetical

judgements. This is expressed by saying for the universal judgements 'If such-and-such conditions are found such a result will follow' (affirmative), 'will not follow' (negative); and for the particular judgements 'If such-and-such conditions are found such a result may follow' (affirmative), 'may not follow' (negative). Or we may use as alternative forms 'If such-and-such conditions are found such a result always follows' (univ. aff.), 'never follows' (univ. neg.), 'sometimes follows' (part. aff.), 'sometimes does not follow' (part. neg.).

(4) That as the alternatives in a disjunctive judgement may not be mutually exclusive we cannot deny one on the ground that the other is true. But we can affirm that if one of the two be false the other must be true.

(5) The distinction between affirmative and negative judgements is called one of *Quality*; that between universal and particular judgements one of *Quantity*; that between categorical, hypothetical, and disjunctive judgements one of *Relation*.

3. Analysis of Judgements.

(i) ANALYTIC AND SYNTHETIC ASPECTS.—A categorical judgement affirms or denies a fact. The fact is given in experience as one; the judgement takes it to pieces, and finds in it two ideas which it at once distinguishes and binds together. For example, the burning fire is one experience; the judgement analyses it into the fire and the burning. Not, as has already been said, that the fire can be fire and not burn; but that in other of our experiences the fire has been for us the source of comfortable warmth or of light.

Similarly, we may have been burnt in other ways than by fire, as, through a burning-glass. Or, at least, we have seen burning due to such other means. It is because what we call things and actions come into our experience at different

times in different relations that we can thus separate in thought what is not separate in reality.

A judgement, then, is, first of all, an act of analysis. But, at the same time, in expressing a particular experience it synthetises, or binds together, the elements it distinguishes.

(ii) SUBJECT AND PREDICATE.—We know that 'Fire burns,' 'Fire cooks food,' 'Fire warms us,' express different total experiences, but we can find in each of them the action of the same thing 'fire.' Of the fire we predicate, or assert, something different in each different judgement we make about it.

It thus appears that there are two parts in every categorical judgement. These are called, as in grammar, *Subject* and *Predicate*. No matter how complex may be the structure of a proposition, or in how many words it may express the judgement, there are always just a subject, and a predication made about it. Until we have distinguished these we have but a hazy notion of what the judgement is.

The subject is the point in the experience from which our thought starts, and that is determined by the interest of the moment. We may be comparing the odours of different roses, and then 'odour' is the subject of each of the judgements we make, and the names of the varieties of roses examined find a place in the predicate.

Of course, when isolated propositions are offered us we have to be guided largely by grammatical considerations. We assume that the subject is probably stated first, or that the name of the thing, as representing the solid and permanent object, is the subject. Or we ask to what question the proposition may be taken as an answer.

In actual life, however, we have not to deal with isolated propositions. Each judgement is a kind of cross-section of the ever-moving stream of thought, and which is subject and which predicate is determined by what has gone before or by what it is anticipated will come after.

For example, suppose I were asked 'What is this rose?' and I replied 'That is a Hugh Dickson,' the predicate would be 'Hugh Dickson,' for that gives the information sought. But if in talking about roses with a friend I had expressed admiration for the Hugh Dickson, and he had said he did not know that variety and asked me to describe it, the best answer I could give would be to point to one and say '*That* is a Hugh Dickson,' when the 'That' and the showing the flower together form the predicate to the subject 'Hugh Dickson.'

In disjunctive propositions it is well to set forth explicitly each alternative as a categorical proposition, and then to decide between subject and predicate.

Hypothetical judgements are not analysable into subject and predicate. Each of the two parts is in form a categorical judgement, but they are thought as unanalysed wholes. The position in which they are thought and expressed in propositions gives them the appropriate names *Antecedent* and *Consequent*.

CHAPTER IV.

THE NORMAL LOGICAL FORM OF PROPOSITIONS.

1. The Normal Form.

In order that the formal relations between categorical propositions may be more easily examined it is customary, before beginning such an examination, to separate the subject from the predicate, and to insert between them *is* or *are*, adding a *not* when the judgement is negative. This link is called the *Copula*. It is always in the present tense in order that when two or more judgements are compared no doubt may arise as to the reference of them all to the same time. Ambiguity would evidently be possible if the copula were either in the past or in the future tense, for time extends indefinitely both before and after the present moment.

This use of a single verb by no means implies that formal logic ignores the existence of all verbs except 'to be.' By no means. Nor does the use of *is* or *are* reduce all relations of fact to simple co-existence. What the copula means is that 'So-and-so can be predicated of such-and-such a subject.' What the predication is remains unaffected. Using S to denote the subject, and P the predicate, we now generally write 'S is P,' but the older form was 'P may be predicated of S'; and the more modern form must be regarded as the equivalent of this.

Such reduction often puts the judgement in an uncouth form, but its aim is convenience, not elegance. It may be likened to the statement of concrete mathematical problems

as equations. It exhibits in the barest form the relation of predication, just as an equation sets forth that of equality. Each is a convenient symbolic mode of expression when the consequences of those relations *only* are to be considered. And each is nothing more.

The symbolic forms of categorical propositions, then, are—

Univ. Aff.	*All S's are P.*	Univ. Neg.	*No S is P.*
Part. Aff.	*Some S's are P.*	Part. Neg.	*Some S's are not P.*

2. Terms of Propositions.

(i) Distinction of Terms.—Subject and predicate are thus set apart, and may be considered separately. At the same time, the copula affirms them to be so connected that the latter may be asserted of the former. They form the two ends or terminations of the proposition, and so are technically known as *Terms*.

In ordinary speech the word 'term' is used more widely, but in the language of logic a term is either the subject or the predicate of a categorical or disjunctive proposition. The hypothetical proposition has no terms, for though each of its two parts is by itself categorical in form, yet each is taken as an indivisible whole, and not analysed.

It is evident that a term may consist of any number of words, provided they are united to express a single idea. It can also easily be seen that certain parts of speech, such as adverbs, conjunctions, prepositions, interjections, cannot stand alone as terms. It is true we can assert 'If' is a conjunction; 'Of' is a preposition; but then the 'If' and the 'Of' are names of the words, and so are used as nouns: the one has no conjunctive, the other no relational, force. But any word can enter into a term composed of several words, as in "Dullness turned up with temerity is a livery all the worse for the facings."[1]

[1] Sydney Smith: *Peter Plymley's Letters*, Lot. 10.

It is evident, too, that when an adjective is used by itself as a term it implicitly refers to the things contained in the universe of discourse. Thus, in 'The English soldiers are brave' the 'brave' implicitly refers to soldiers, not to Antarctic explorers or hunters or lions or dogs. We may, if we will, insert the noun, and the proposition then emphasises that in the wider class of 'brave soldiers' English soldiers are included. But that is only its secondary signification. The primary meaning is the direct attribution of a certain kind of bravery—military bravery—to English soldiers. So we see that a judgement is primarily the attribution of a predicate to a subject, and secondarily the inclusion of the subject in the class of things named by the predicate.

(ii) Denotation and Connotation of Terms.—This draws our attention to the fact that most terms can be looked at from two points of view; or, in other words, that either of two aspects of their meaning may be emphasised.

A name means first of all the things to which it can be applied. That is how we learn to use names. And often we know little more of those things than is sufficient to enable us to identify them.

Yet there is always a reason why a thing receives its name, and so is classed with all other things to which that name belongs. While our knowledge is small we go by superficial resemblances; but as it increases we see more deeply, and at times have to revise our former judgements as to the class to which an object belongs, and, consequently, the name that should be given it. For example, whales were formerly thought to be fish because they live in the sea, and sponges were regarded as vegetables because of their appearance. Increased knowledge of the structure and mode of life of each has shown that the former is not a fish but a mammal, and the latter not a vegetable but an animal.

As was seen in the second chapter, the amount of know-

ledge people possess of the things they can identify varies enormously with individuals, according to the amount of attention each has paid to them. That we do not regard this personal knowledge as an ultimate standard is shown by the readiness with which we accept the assurance of biologists that whales are not fish nor sponges vegetables. So that, though we may not know exactly what it is, we believe there is a normal or legitimate meaning, which determines with authority the correct application of names. This is called the meaning in *Connotation* or *Intention*, while the range of application is named the meaning in *Denotation* or *Extension*.

(*a*) *Meaning in Connotation.*—When the meaning in connotation is explicitly set forth we have a scientific definition of the word; that is, a statement of the test by which it can be conclusively determined whether the name defined should be applied to particular things, or groups of things. This seldom corresponds with the so-called definitions in dictionaries. Their purpose is to enable us to recognise the things, and to apply the names correctly. Often a brief description aided by a picture is the readiest and surest way of doing this. Such definitions refer primarily to denotation.

Knowledge of scientific definition or connotation is exceptional with us all. Such definitions result from long and patient inquiry into the nature of things, and each one marks a stage in the growth of exact knowledge. But none is regarded as necessarily final. Further knowledge, or a fresh conception of what is most essential, may lead to the revision of a definition.

For every-day practical use a fairly correct knowledge of denotation is the one thing needful. It enables us to refer clearly to the things of which we are thinking, and so to convey our thoughts intelligibly to others. To this, each of us adds as much pertinent knowledge as he possesses, and it

is mainly because this knowledge is not the same for all of us that we so often misunderstand each other.

(*b*) *Relation of Connotation and Denotation.*—We systematise our knowledge not only by thinking the things of our experience as belonging to classes, each marked by its own name, but by dividing the wider classes into narrower, and those into classes yet more limited. And each of these has its name.

Evidently the name of a widest class must denote every member of each of its sub-classes, and cannot, therefore, include in its connotation any of the marks which distinguish those sub-classes from each other.

For example, the name 'ship' covers all vessels that move on the waters, whether propelled by steam or by the wind, whether rigged as barques, brigs, schooners, or in any other way. Neither mode of propulsion, then, nor rig of sail, can enter into its connotation, for that would exclude from the denotation of 'ship' objects that can rightly claim the name. Formerly we could stop there, but the invention of submarines and the naming a kind of dirigible balloon 'air-ship' have further increased the denotation. This extension cuts out from the connotation all reference to floating on the water.

The very marks which distinguish any one sub-class from its fellows, and are, therefore, excluded from the connotation of the name of the wider class, must, for the same reason, be part of the connotation of its own name.

'To have two masts' is part of the connotation of both 'brig' and 'schooner,' necessary to mark them off from vessels with a different number of masts. To distinguish them from each other, and from brigantines and other forms of two-masted ships, the rig of each mast must be added. Thus, a 'brig' is a two-masted vessel square-rigged on both masts, a 'schooner' one fore-and-aft-rigged on both masts, a 'brigantine' one with square-rigged fore-mast and fore-and-aft-rigged main-mast.

We see, then, that the denotation of the name of a wider class is greater than that of any of the sub-classes contained under it; and, on the other hand, that the connotation of the name of each narrower class must add to that of the wider class the characteristics which mark it off from the other sub-classes made by that division. So in a series of acts of classification the connotation increases, and the denotation decreases, as we pass from the widest, through the intermediate, to the narrowest class.

The limit is reached when the term denotes only one individual, as 'The schooner with her hull painted black with a red stripe, now lying opposite the most southerly crane on the quay.' Here the connotation has swollen far beyond that of 'schooner,' as each of the characteristics, such as 'painted black,' can be shared with other schooners, and, consequently, can form the basis of a sub-class of schooners. For we can make any sub-class we please out of a wider class, provided we find a common but distinctive characteristic to add to the connotation of the name of the wider class.

(c) *Proper Names.*—Suppose the schooner to be named the 'Eliza.' The same object is denoted, but the Proper Name does nothing more. It implies no characteristics, and consequently cannot be defined.

Of course, to anybody who knows the vessel and its name, 'the Eliza' will convey just as much meaning as the rather lengthy term we used before. But we have seen that what a word means to you or me is not its connotation. That is meaning which can be expressed in a definition; that is, which the word by itself implies. But a Proper Name is only a convenient verbal label, and means no more than does a label on a portmanteau. Each serves as a mark of identification; but the Proper Name no more implies the characteristics of the person, place, animal, or thing, to which it may be applied, than the luggage-label gives informa-

tion as to the contents of the portmanteau. You or I may know all about the matter in either case, but others might see the label, or hear the name, and be none the better informed.

Proper Names, then, have no connotation, and the denotation of each is just one. For though the Smiths and Joneses of the world may be legion, each has his own label.

(iii) Singular and General Names.—This leads us on to notice a grammatical distinction introduced into logic at a time when it was held to deal chiefly with language. Terms were classed into those that apply to only one object, when they were called *Singular* or *Individual*, and those that apply to a class composed of a plurality of things.

This is not a logical question at all. A class may include any number, and there is no more reason to deny the possibility of classes of one than of classes of two. Indeed, the use of 'Some,' as the formal mark of indefinitely known quantity, to cover all cases in which we cannot say 'All,' shows that to make this a distinction of *terms*—i.e. elements of a proposition—is inconsistent with the distinction between universal and particular propositions; for that depends on the quantity of the subject. A singular term exhausts the denotation of its class, and a proposition with such a subject is, therefore, universal; for the scope of its application is as definitely stated as is that of a general term by the use of such words as 'All' or 'Every.'

Collective Names.—Sometimes *Collective Names, i.e.* those which denote groups of things, are treated as a further class of *terms*. This is another grammatical distinction.

For logic, a collective name is like every other general name. It denotes things, though they are composite things, made up of objects we may equally well consider separately. An army is composed of corps, a corps of divisions, a division of battalions, a battalion of regiments, a regiment of companies, and a company of men. It is only a special case of

continued partition. We can think of any group, either as a unit by itself, or as forming part of a larger unit. So, each word can be used in both the singular and the plural number. The important distinction is that between the collective and the distributive use of terms.[1]

In another use, the collective name is a convenient abbreviation for 'all the members of the group.' Then *they* are the real subject of the judgement, and this should be indicated by the use in the proposition of a plural verb.

The current disregard of distinctions of thought and language has of late years led to much neglect of this protection against ambiguity. We read such statements as 'The cabinet have decided so-and-so,' when the decision was evidently not that of the separate individuals but that of the whole body collectively. When in any meeting a decision is reached by a vote, such sentences as 'The committee are of such-and-such an opinion' is not only misleading, but actually false, for the words mean that *all* the members were of that opinion.

To take another example: 'The class was examined' means that the class was examined, and its work gauged, as a whole. But 'The class were examined' signifies that each member was tested individually, and that the results were estimated separately. Of course, an average could then be struck. But only in an unnatural sense can that be called 'the work of the class,' for that phrase implies co-operation, and when "brother helps brother" each often rises to heights of intelligence which unaided he could not reach.

(iv) Positive and Negative Terms.—The most common mode of denial is by a negative proposition. But, for examination of some of the formal relations of propositions, it is sometimes convenient to transfer the 'not' from the copula to the predicate, and thus obtain a proposition,

[1] See pp. 34-35.

affirmative in form, though retaining its negative meaning. Thus, S | *is not* | P may be written S | *is* | *not-P*, and this new symbolic form may be substituted for the original one whenever it is more convenient. It has no significance beyond facilitating the manipulation of symbols. The retranslation of the symbols into real terms will not be affected.

The importance of certain qualities, and their frequent absence, have together led to the use of pairs of terms, one of which is formed from the other by the addition of a negative prefix or suffix; as, equal, unequal; happy, unhappy; honest, dishonest; joyful, joyless.

A little examination shows that there is not a uniform relation between these. 'Equal' and 'unequal' leave no place for an intermediary: they exactly fulfil the conditions of the Principle of Excluded Middle. But between the members of many such pairs there are intermediate possibilities. We cannot conclude that because a man is not happy he is unhappy; for between those terms are states of mind which we should not call by either name.

Most negative terms in common use have gradually acquired a greater or less amount of positive meaning; *e.g.* 'unhappy' implies not merely absence of happiness but presence of some actual misery. We often have series of terms which more or less inadequately express the continuous way in which one experience shades off into its opposite. As between black and white there are innumerable shades of grey, so between ecstatic happiness and the most abject misery there are innumerable gradations. Each of us determines for himself where the positive 'happy' and the positive 'unhappy' begin, and we do not always decide it in the same way. Nor can we always be sure on which side the dividing line we should place our actual state.

We must, then, be careful to guard against the assumption that the 'negative' terms of every-day use have only a

negative meaning. When we want to apply the Principle of Excluded Middle we should make a formal negative by prefixing 'non-' or 'not-' to the positive term; or, better still, perform all such operations with symbols.

(v) ABSTRACT AND CONCRETE NAMES.—The traditional distinction between *Concrete Names*, or those which denote things, and *Abstract Names*, or those which refer to qualities considered apart from things, is of purely linguistic import. Adjectives, such as 'strong,' 'wise,' are names of all things which possess those qualities; but the abstract names 'strength,' 'wisdom,' refer to ideas in our minds which we have formed from comparing strong and wise acts in contrast with weak and foolish ones.

The distinction not only has no logical value, but is misleading, for it obscures the important fact that, as all thought analyses experience, all thought is abstract. The distinction between what is concrete and what is abstract is this: the concrete is the fact in all its complexity; the abstract is any aspect of that fact. For the fact as a whole we have artistic or moral appreciation; but immediately we set ourselves to try to understand it we have to fix our attention first on one element or aspect, then on another. Thus, every act of thought—that is, every judgement—is abstract in a lower or higher degree. Abstraction is not a single dealing with fact. The deeper we probe into its nature, the more abstract our thought becomes; that is, the further it is from its starting-point in the concrete fact.

For instance, such a judgement as 'William the Conqueror was the first Norman king of England' is but a small step from the concrete. But in that it states only one aspect of William's life, it shows the beginning of abstraction.

Similarly, 'The sun is very warm' expresses something which we directly experience. But it is only a part of the experience. The sun is also seen to be bright, to be in a definite part of the sky, etc All these together make the

concrete fact; but every judgement about it is an abstract statement of one of its aspects. The process of abstraction advances till we form judgements about such universal aspects of all matter as attraction.

The consideration of all this is of value for the theory of knowledge, but it has no bearing on the processes of inference, and so, even this true and important distinction does not properly belong to logic.

(vi) Absolute and Relative Names.—An *Absolute Name* is one which does not imply any specific relation, as 'man': a *Relative Name* is one which does imply such a relation, as 'father' implies 'child'; 'friend' implies 'friend'; 'foe' implies 'foe.'

This traditional distinction also is one of little logical significance. We will only note that, as the same relation can be stated from both ends, the passage from the one statement to the other is not an inference of a new fact, but merely a change in form of statement. The fact that 'The Prince of Wales is the son of the king of England' is not inferred from 'The king of England is the father of the Prince of Wales'; it is the same fact expressed in an alternative form. The difference is rhetorical. In one context one form is preferable; in another context the other better brings out the point of view we wish to emphasise.

8. Reduction to Normal Form.

(i) Value.—In actual speech judgements are expressed in all kinds of ways, and with various degrees of exactness. The reduction of the sentences of common life to the form subject + copula + predicate often demands much care and acuteness to avoid changing the assertion when we change the form. It cannot be done correctly unless we make quite clear to ourselves exactly what judgement the statement before us conveys.

We must regard every sentence as a system of words re-

presenting a system of thought, and our task is, through examination of the relations of the former to discover those of the latter. The practice is, thus, a very profitable one, in that it insensibly builds up a critical attitude towards speech, and a demand that its expression of thought shall be clear and precise. The power to see exactly what a statement really asserts, when stripped of the adornments with which rhetoric may have decked it, is the first qualification for clear thought, and consequently the first step in the practical study of logic. We strongly urge the reader to give time and care to reducing statements gathered from books and newspapers to this bare form, which brings into the clear light of day exactly what assertion is made.

(ii) Rules.—Like all analysis, that of sentences should be conducted methodically, and should be exhaustive. The following rules of procedure may profitably be followed—

(1) Find the subject; *i.e.* the nucleus of the thought.

(2) Determine whether the judgement is universal or particular.

(3) Separate copula from predicate; *i.e.* if the predication is made by any verb other than 'is' or 'are,' change it into an equivalent phrase beginning with one or other of those words. The rest of that phrase is the predicate.

(4) Determine whether the judgement is affirmative or negative.

(5) Determine the universe of discourse, so as to fix the limits of the assertion.

(iii) Examples.—We will now analyse a few typical examples, as to see a thing done is more profitable in suggestion than merely to be told how it should be done.

(*a*) 'Happy is the land that has no history.'

The sentence is rhetorically inverted, and (1) the subject is 'The land . . . history.' The relation is assumed to be general, so (2) the judgement is universal: we write 'Every

land.' (4) The judgement is affirmative, and (5) the universe is 'land.' The formal statement is

'Every land that has no history | is | a happy [land].'

(*b*) 'All is not gold that glitters.'

(1) The natural order is the Shakespearean 'All that glitters is not gold.' (2) The apparently universal character of the proposition is rhetorical; the real reference is to 'some glittering things,' which are (4) denied to be gold. (5) The universe is 'things.' Result—

'Some glittering things | are not | golden [things].'

(*c*) 'None of the audience failed to see the point.'

At first sight the judgement looks negative, but there are two negations in the meaning which really neutralise each other. The negative form is, then, only rhetorical, and the plain judgement is 'Every member of the audience saw the point.'

(1) and (2) are obvious, (3) 'saw the point' becomes 'is a person who saw the point,' and (5) this gives the universe. Result—

'Every . . . audience | is | [a person who] saw the point.'

(*d*) 'Quill pens are the only ones I care to use.'

Consideration of the force of the statement shows that it is not made about 'pens' but about my personal preferences. So, (1) the subject is 'pens I care to use.' (2) 'Only' shows that the judgement is universal; (5) the universe is 'pens.' Result—

'All the pens I care to use | are | quill [pens].'

(*e*) 'Both theory and experience prove that unjust laws provoke discontent.'

Two judgements are here united, as the assertion could be made about both theory and practice apart from each other.

But the union adds strength, and it may well be that this very union is the subject about which the assertion is made. Only the context could definitely determine this, but taking this as the meaning, (2) the quality is universal, for the relation is given as a general one. (3) The predicate as given is changed into 'is a proof of discontent.' (5) The universe is 'events having probative force.' Result—

'Every instance of both theory and practice | is | [an event] proving that discontent attends unjust laws.'

(*f*) 'Neither force nor flattery can alter my decision.'

(5) The universe is 'means of altering my decision.' The judgement is (2) universal, and (4) negative. (3) The predicate is changed into 'is capable of altering my decision.' Result—

'No [means included in] force and flattery | is | [a means of] altering my decision.'

(*g*) 'None but the brave deserve the fair.'
(*h*) 'Only the brave deserve the fair.'

The difference is wholly rhetorical. The question raised is as to who are worthy of the fair; the answer is 'only the brave.' (1) The subject, therefore, is 'persons worthy of the fair.' (2) As indicated by 'none' the quantity is universal. (4) In (*g*) the negation is neutralised by 'but,' and, as in (*h*), the judgement is affirmative. (3) The copula is implied in 'deserve' and in the formal statement becomes 'are deserving,' of which 'deserving' belongs to the subject; the predicate is 'brave.' (5) The universe is 'persons,' or, if it be preferred, 'men.' The formal statement is—

'All [persons] deserving the fair | are | brave [persons].'

(*i*) 'Nothing can be more contemptible than the man who dedicates all the energies of his mind to the indulgence of his appetites.' (*Wm. Godwin.*)

(1) The assertion is about a type of man; the sentence is inverted for the sake of rhetorical emphasis. (2) It is universal, as is shown by the exclusive 'nothing.' (3) 'Nothing . . . than' can be changed into 'is a person worthy of the greatest possible contempt.' (4) The judgement is affirmative. (5) The universe is 'persons.' Result—

'Every man . . . appetites | is | [a person] worthy . . . contempt.'

(*j*) 'The man that is not provided for to-morrow cannot enjoy to-day.'

(1) The subject is 'man unprovided for to-morrow.' (2) The judgement states a universal relation, and so the negative in 'cannot' becomes 'no' as the sign of quantity. (3) 'Can enjoy' becomes 'is able to enjoy.' (4) The judgement is negative. (5) The universe is 'men.' Result—

'No man unprovided for to-morrow | is | [a man who is] able to enjoy to-day.'

(*k*) 'I am sure we are not to be saved by religious hatred, and by religious trifling.' (*Sydney Smith.*)

It would be possible to take this as primarily intended to state a personal opinion, when all that follows 'am' would be the predicate. But even when the sentence stands alone this is not the most probable interpretation, and reference to the context makes it certain that the 'I am sure' is only a rhetorical device for making the real assertion more emphatic. The judgement, then, deals with the bearing of 'religious hatred and trifling' upon the safety of England during the Napoleonic wars. The passive form suggests an inversion of the order of subject and predicate, and that is borne out by the sense. So, (1) the subject is 'religious . . . trifling.' The proposition is (2) universal, for no limitation of scope is

implied, and (4) negative. (5) The universe is 'means of national safety.' Result—

'No religious . . . trifling | are | [means to our national] safety.'

(*l*) 'Promises, like piecrusts, are made to be broken.'
(*m*) 'Trespassers will be prosecuted.'
(*n*) 'Astrology makes the stars a cause or sign of jealousy.' (*Burton.*)
(*o*) 'Right-angled triangles are inscribable in semi-circles.'

The chief question raised by each is whether the judgement is universal or particular, for no sign of quantity is given. The decision must depend upon whether the relation between the predicate and the subject is universal and necessary.

In (*n*) and (*o*) this is so, for

(*n*) interpretation of the influence of the stars on human life was of the essence of astrology; and

(*o*) the geometrical relation can be proved necessary by rigid reasoning.

Each of these judgements is, therefore, generic,[1] and the formal statements are—

(*n*) 'All astrology | is | [a pseudo-science which] makes the stars . . . jealousy.'

(*o*) 'Every right-angled triangle | is | inscribable in a semi-circle.'

But in (*l*) and (*m*) there is no such necessary relation, but only occasional and accidental ones. Those judgements are, therefore, particular.

(*l*) The breaking of promises is not implied in the making of them, but experience teaches that the one sometimes follows the other. The formal statement is—

'Some promises | are | [engagements] made to be broken.'

[1] See p. 45.

(*m*) The familiar statement about trespassers is really an abbreviated way of stating that trespassers are liable to prosecution. The reference is to a system of law, and, in that, it may be true that 'All trespassers are liable to prosecution.' But as a statement of the risk incurred by individual trespassers it can only be regarded as particular. For, obviously, only those *found* trespassing can be prosecuted, and it is well known that legal action does not always follow discovery. It may, then, be stated—

'Some trespassers are [persons who will be] prosecuted,' which is the formal categorical expression of the modal proposition 'Any trespasser may be prosecuted.'

(*p*) 'Few boys are fond of study.'

This definitely excludes an indefinite number of boys from those who like study. So the normal form is

'Some boys are not fond of study.'

(*b*) 'A few boys are fond of study.'

Here, on the other hand, the positive assertion is made of an indefinite number of boys. So the normal form is

'Some boys are fond of study.'

CHAPTER V.

FORMAL RELATIONS OF PROPOSITIONS.

1. Relations between Categorical Propositions.

Suppose you asserted 'All books on logic are dull.' How could I convince you that you were wrong? To contradict you in words would appear rude, and at best would only show that my opinion was at variance with yours. But if I could produce a single treatise on the contemned subject which you could not call dull I should have contradicted you in fact. Then, suppose I were able to show several more, among the many thousands of books on logic which have been written, to which the epithet could not reasonably be applied, you, as a just person, would be led to admit that 'Some books on logic are not dull.' If, however, in my enthusiasm I were bold enough to meet your first wholesale condemnation by the equally comprehensive assertion 'No books on logic are dull' your task of convicting me of error would, I fear, not be a difficult one. Soon the incontrovertible evidence you would advance would compel me to acknowledge that 'Some books on logic are dull.'

(i) Contradiction.—Examination of this example shows that all four of the forms in which categorical judgements can be expressed have been used, and that they divide into two sets, each consisting of two propositions which exactly exemplify the Principles of Contradiction and Excluded Middle, in that if either be true the other must be false, and if either be false the other must be true. Such a relation is called *Contradiction*. It will be noted that in each pair of

contradictories the two propositions differ both in quality and in quantity; so that they have the greatest possible amount of formal unlikeness. Contradiction, then, holds between a universal proposition of one quality and the particular proposition of the opposite quality.

Of course, the reference of both the terms must be identical in the two propositions: there is no contradiction between 'All books on logic are dull' and 'Some novels are not dull.' The contradiction exists when the 'some' of the particular proposition is identically the same as an indefinite part of the 'all' in the universal proposition, so that of the *same* objects the assertion and the denial of dullness are simultaneously made. Then, of necessity, one must be false, the other true.

Nor would there be any contradiction were 'dull' in the one proposition to refer to an intellectual quality, and in the other to the colour of the binding. The dullest book in either sense may be far removed from dullness in the other.

(ii) CONTRARIETY.—In the next place, we see from the example that, though it was likely to be comparatively easy to contradict—or disprove the truth of—either of the universal propositions, it would be a very different thing to establish the truth of the other universal. Any universal is overthrown by a single contradictory instance. So it holds its place by a very precarious tenure, unless no such instances can possibly exist, and that is only when the universal proposition expresses a generic judgement.[1]

In all other cases, where the 'every' is really limited to the range of one's own experience and the testimony of trustworthy witnesses, the finding of an adverse instance is not unthinkable. That 'all swans are white' long stated an uncontradicted experience. When Australia was discovered there were found birds of the same species, but of a black colour. Had they been given a different name, the old uni-

[1] See p. 45.

versal judgement of experience would still have been true. But as they were classed as 'swans' that judgement in its unlimited form was overthrown, though 'All swans of European and Asiatic origin are white' might still be maintained.

Two universal judgements, then, one affirmative, the other negative, referring to identically the same matter, make assertions as far removed from each other in fact as is possible. They are, therefore, called *Contraries*.

If either be true the other must be false; but to prove that one is false does not at all establish the truth of the other. Both may be equally false; as is, indeed, very often the case with judgements simply based on experience. Many people both generalise rashly, and imagine that the wider the range of an assertion the greater is its strength. Consequently, many results of experience, that if expressed in particular propositions would be incontrovertibly true, are put in the universal form, which, as we have seen, is much more easily disproved.

Contrary judgements, then, are not so closely related as are contradictories. They differ in the relation of quality only, not in that of quantity. So the inference from the truth of one to the falsity of the other cannot be reversed. The opposition between them is not so complete *in form* as with contradictories, though they are further apart *in fact*. The Principle of Contradiction is exemplified, but not that of Excluded Middle.

(iii) SUBALTERNATION.—We may now examine the relations between propositions differing only in quantity; that is, between a universal and a particular, referring to exactly the same matter, when both are affirmative or both are negative.

As the relations are of quantity only they will hold equally in affirmative and in negative pairs. It is obvious that if 'All books on logic are dull' be true, the indefinite 'Some books on logic are dull' must be true also; for the 'some books' are

included in the 'all books.' It is equally clear that if the particular proposition cannot be truly asserted, still less can the universal be affirmed.

On the other hand, though dullness may be truly asserted of 'some' it cannot be assumed that it will hold of all. In the 'all' may be included two kinds—those that are, and those that are not, dull. It follows from this that the assurance of the truth of the particular proposition gives us no ground for asserting the universal. The aim of thought is to advance from indefinite particular assertions to definite universal judgements, but this often involves limiting the range of the subject, as in the case of the judgement about the colour of swans.

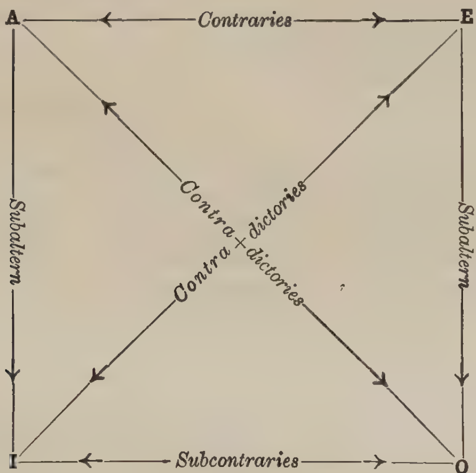
Gathering together the results, we see that (1) the truth of the universal involves the truth of its subaltern—*i.e.* of the particular of the same quality; (2) the falsity of the particular involves the falsity of the universal of the same quality.

(iv) SUBCONTRARIETY.—It only remains to consider the relations between particulars of opposite qualities referring to identically the same matter. As each is quite indefinite in the range of its application, both may be true together. Some books on logic may be dull, and others not dull. But both propositions cannot be false, for then there could be no books on logic in existence, and so neither form of words would be a real proposition, as it would not express either an actual or a possible judgement.

(v) SYMBOLIC EXPRESSION.—It is a convenient saving of time and space to use symbols to represent the kinds of propositions. Traditionally, the first two vowels of the Latin *affirmo* have long represented the affirmative, and the two vowels of *nego* the negative, propositions. So

For *All S's are P* we write **A**. For *No S* is *P* we write **E**.
For *Some S's are P* we write **I**. For *Some S's are not P* we write **O**.

It is also traditional to symbolise their formal relations by the appended diagram, known as the *Square of Opposition.* All the relations are technically called relations of 'opposition', because in the figure the symbol for each member of every relation stands at the opposite end of one of the lines to the other member of that relation. Opposition, in the non-technical sense of incompatibility, only exists between contradictories and contraries.



The universals are at the top corners, and the affirmatives at those on the left hand side.

That contradiction is the most complete formal opposition is marked by the diagonals as the longest lines, while the subaltern particulars receive their name from their position in the diagram beneath the universals of the same quality.

Similarly, the term 'subcontrary opposition' means simply that relation which is written *under* the contrary relation in the square.

(vi) SUMMARY.—We may sum up—

A and **O**; **E** and **I**: pairs of contradictories. In each pair truth of either member involves falsity of the other; and falsity of either involves truth of the other. One must be true, the other false.

A and **E**: contraries. Truth of either involves falsity of the other. But both may be false.

A and **I**; **E** and **O**: pairs in which the particular is subaltern to the universal. Truth of universal involves truth of particular: falsity of particular involves falsity of universal. But the converse inferences do not hold.

I and **O**: subcontrary relation. Both may be true; both cannot be false.

Or, more generally: "The affirmation of a universal proposition, and the denial of a particular one, enable us to affirm or deny all the other three; but the denial of a universal proposition, and the affirmation of a particular one, leave us unable to affirm or deny two of the others." [1]

(vii) OPPOSITION OF SINGULAR PROPOSITIONS.—The relations we have considered hold between propositions in which the subject is a general term used distributively. When the subject is a singular term, or, what has the same force, a general term used collectively, the distinctions of quantity are absent, and there is possible only the affirmation and denial of the same fact. The contrary is, as it were, absorbed into the contradictory. Hence the importance of distinguishing carefully between these two possible uses of the same term.

'All the present taxes (together) are impossible as permanent means of raising revenue' does not mean that not one of them can be retained, but that they cannot be retained in the whole mass. It is, therefore, not contradicted by

[1] De Morgan: *First Notions of Logic*, p. 8.

'Some of the present taxes are not impossible as permanent means of revenue,' for it is in no way incompatible with it. The 'All the present taxes' is considered as one burden on the tax-payers, and has, therefore, the force of a singular term. It is contradicted by denying that this whole mass of taxation is insupportable; that is, by asserting that 'All the present taxes (together) are not impossible as permanent means of revenue.'

2. Relations between Hypothetical Propositions.

As there can be hypothetical propositions corresponding to all the four forms of categorical proposition, the relations of opposition can be applied to them. Here, too, contradiction and contrariety are the most important relations, and subalternation is of value only so far as the particular suggests the search for the universal.

Using **X** and **Y** to represent the categorical propositions of which both antecedent and consequent consist, as: 'If experience has taught us anything (**X**), it is the impossibility of perpetual blockades'[1] (**Y**): we may write the four forms symbolically, using $\equiv$ to mean 'is equivalent to'—

Univ. Aff. ($\equiv$ **A**): *If* **X** *is true, then* **Y** *is always true.*
Part. Aff. ($\equiv$ **I**): *If* **X** *is true, then* **Y** *is sometimes true.*
Univ. Neg. ($\equiv$ **E**): *If* **X** *is true, then* **Y** *is never true.*
Part. Neg. ($\equiv$ **O**): *If* **X** *is true, then* **Y** *is sometimes not true.*

3. Relations between Disjunctive Propositions.

All disjunctive propositions are affirmative, for they assert that a choice of predications exists. But they may differ in quantity, and may thus correspond to the categorical **A** and **I** forms. The square of opposition may then be completed by propositions contradictory and contrary to **A**.

[1] Sydney Smith: *Peter Plymley's Letters*, Let. 7.

For example, we may have as the original propositions

(**A**) 'Every bald man either wears a wig or easily catches cold.'

(**I**) 'Some bald men either wear wigs or easily catch cold.'

The contradictory of the former is 'Some bald men neither wear wigs nor catch cold easily' (**O**), and its contrary 'No bald men either wear wigs or catch cold easily' (**E**).

But neither of these propositions is disjunctive, for to deny an alternative is not to offer another one.

4. **Distribution of Terms.**

Each of the four normal types of categorical proposition makes it clear whether the subject refers explicitly to everything that bears the name, or whether the extent of the reference is indeterminate. No corresponding sign of quantity is affixed to the predicate, because it directly names qualities, and only indirectly the things which possess them. Nevertheless, the predicate, as well as the subject, has denotation. Indeed, we have seen that determination of the universe of discourse involves reference to the denotation of both terms.

Let us, then, consider what the normal types of proposition imply, *by their mere form*, as to the scope of the application of their predicates.

No inference from outside knowledge of the actual application of the terms of any special proposition taken as an example is here to the point. The question is not to what any one judgement may, in point of fact, refer, but to what every proposition of that form *must* refer. Only on this basis can we legitimately substitute one form of proposition for another, as being formally implied by it; and to do this is often convenient.

As the terms 'universal' and 'particular' are applied to propositions, it would be confusing to speak of universal and particular terms. So, *distributed* and *undistributed* are

traditionally used instead. The choice is not a happy one, for confusion may arise between a distributed term—*i.e.* one used in the whole range of its application, and a term used distributively—*i.e.* as opposed to collectively.[1] Care must be taken to keep these clearly distinguished.

As the distribution of the subject cannot affect that of the predicate, there are only two cases to consider—the affirmative and negative forms of proposition.

In the affirmative, to assert that *All*—or *Some*—*S's are P* means that *P* can be asserted of them, but leaves it quite open whether *P* can also be affirmed of other things. In other words, the *All*—or *Some*—of the things known by the name *S* are to be found among the things known as *P*. That is to say, the whole, or part, of the class *S* is included in the class *P*, but whether it wholly fills that class, or leaves a *not-S* surplus, is left an open question.

In 'All thinking is hard work' the forms of hard work are not confined to thinking. In 'All triangles are three-sided figures' the two classes are identical in fact. In 'Some persons are logicians' our knowledge of the matter may suggest that all logicians may in fact be included among the 'some persons.' But for 'persons' substitute 'Members of Parliament,' and the suggestion is quite otherwise.

We have given these examples, therefore, not to illustrate the rule of distribution, but as indications of the facility with which material examples may put us off the scent when we are hunting for a formal law; that is, a law which holds in every case of that form, because it results from the form alone.

Formally, then, *in every affirmative proposition the predicate is undistributed*: the implicit sign of quantity is 'some.'

On the other hand, in a negative proposition the exclusion asserted must be total, or it is not exclusion at all. Jones is

[1] See pp. 35, 59.

not precluded from playing football by being excluded from the first team, but only when he is excluded from all the teams. 'No pigs fly' denies to pigs membership of all the classes of things that have the power of flight, not merely of some of them.

Thus, in a negative proposition the things referred to by the subject term—however many they may be—are shut out from the whole denotation of the predicate term. That is to say, *in every negative proposition the predicate is distributed*: the implicit sign of quantity is 'All.'

To sum up the whole matter of distribution—

Universal propositions have distributed subjects.
Negative propositions have distributed predicates.

Combining these—

E	propositions	distribute	both subject and predicate.
I	,,	,,	neither subject nor predicate.
O	,,	,,	the predicate.
A	,,	,,	the subject.

5. Formal Eductions of Implication.

(i) Principle.—We are now in a position to consider what other propositions of the four typical forms are formally implied by each of them. The drawing out of these implications is called *Eduction.*

It is valuable for two purposes: it makes the full force of each form of assertion more apparent; and it allows of the substitution of one form for another, which is as convenient in logical as in algebraic reasoning.

The general principle is self-evident: "If a proposition assert agreement or disagreement, any other proposition which asserts the same, to the same extent and no further, must be a legitimate consequence."[1]

[1] De Morgan: *First Notions of Logic*, p. 9.

The fallacy to avoid is that of extending the range of application of the original proposition.

(ii) Obversion.—By changing the quality of *both* copula and predicate of any given proposition we obtain an exactly equivalent proposition of formally different quality. This is known as *Obversion.*

There is no interchange of terms, and the quantity of the proposition is unaffected. Thus, each of the universal propositions, **A** and **E**, can be obverted into the other; and the same is true of the particular propositions, **I** and **O**. Obversion is a mutual process, and in no case is there any loss in the range of application in the obverse proposition.

As examples we may take—

Obverses of each other	**A**	All unjust laws are opposed.
	E	No unjust laws are unopposed.
Obverses of each other	**I**	Some surgeons are well-paid.
	O	Some surgeons are not ill-paid.

(iii) Conversion.—The question naturally suggests itself: Can the terms of a proposition be interchanged, so that P becomes the subject and S the predicate? Such a process is called *Conversion.*

Evidently, the distribution of the predicate comes into the question, as in the converse proposition that term must be given a definite sign of quantity when it is made the subject.

(*a*) **E** *and* **I** *Propositions.*—Conversion cannot affect quality, so we have only to remember that in **E** propositions both terms are distributed, and in **I** propositions both are undistributed, to see that interchange of terms in these types of proposition will not touch the range of the assertions they make.

To say that S is set apart from every P is exactly the same as to say that P is set apart from every S. If 'No pigs fly' then 'Nothing with power of flight is a pig.'

With **I** it is obvious that S and P are but different names

of the same objects. However few or many the 'Some *S's*' may be, they are the same individuals as the *P's* to which they are equated. If some boys are idle evidently there are some idle people who are those very same boys. There is only a change of name, so that no fresh light is thrown on the matter of the judgement.

E *and* **I** *propositions, then, can be formally converted simply.*

(*b*) **O** *Propositions.*—The subject is undistributed, and, therefore, cannot be made the predicate of a negative proposition. All the four forms of proposition with *P* as subject and *S* as predicate are compatible with 'Some *S's* are not *P*'; the terms symbolised by *S* and *P* being, of course, kept identical. For example, each of the following lines contains two quite compatible propositions—

Some boys are not idle	and	All idlers are boys.
Some boys are not idle	and	No idlers are boys.
Some boys are not idle	and	Some idlers are boys.
Some boys are not idle	and	Some idlers are not boys.

O *propositions, then, are formally inconvertible.*

(*c*) **A** *Propositions.*—In an **A** proposition the predicate is undistributed. Hence, when it is made the subject of the converse proposition that proposition can only be particular. So **A** converts to **I**.

The original proposition—or convertend—and its converse are not, as they are in the case of **E** and **I** propositions, reciprocals, or exact equivalents. The converse is particular, and, therefore, affirms less than the universal from which it is derived. We cannot get back to where we started from by again converting the converse.

This conversion is technically said to be *by limitation*, or *per accidens*. Obviously, it cannot bring out a new aspect of the original judgement. Its sole use is as a permissible step in certain formal processes.

A *propositions, then, are formally converted by limitation.*

(*d*) *Summary.*

> **E** *and* **I** *propositions can be formally converted simply.*
> **A** *propositions can be formally converted by limitation.*
> **O** *propositions cannot be formally converted.*

(*e*) *Fallacy of the Consequent.*—The fact that **A** propositions cannot be converted simply gives opportunity for fallacy of which not a few folk avail themselves.

Suppose a reviewer wrote: "The writer claims that the study of logic helps people to acquire skill in thinking. Does he not see that this is contradicted by daily experience? Do we not all know men—many men, perhaps—who are distinguished by their power of thought, though they have never studied logic?"

"True, my friend," the contemned logician would reply, "we do know such men; but—dare I even hint it?—you here give no proof that you are one of them. For, to say that the study of logic is a road to clear thinking is by no means to deny the possibility of other roads. One may grant that all logicians are critical without committing oneself to the assertion that all critics are logical."

Rhetorical language hides the fallacy, and it is often committed unintentionally. People reason badly with the best intentions. But good intentions avail nothing at the bar of logic, however pertinent they may be at that of ethics.

The formal fallacy consists in this: In the **A** proposition the predicate is undistributed; if we simply interchange the terms we make it distributed; then the new assertion passes beyond the limit of that from which we claim to derive it. Thus, we break the rule of eduction.

We have no right to infer from 'All statesmen are politicians' that 'All politicians are statesmen.' As well educe from 'All Members of Parliament are paid £400 a year' that 'All who are paid £400 a year are Members of Parliament.'

This mode of falling into error Aristotle named the *Fallacy of the Consequent.* He wrote: "If B follows from A we imagine that A must follow from B, . . . because rain wets the ground, wetness of the ground is supposed a proof of rain. . . . This does not follow. . . . Because every man in a fever is hot it does not follow that every man who is hot is in a fever."[1]

These examples would be more appropriately expressed in hypothetical propositions: 'If it rains the ground is wet'; 'If a man has a fever he is hot.' But rain is not the only possible origin of wetness of ground, nor is fever the only cause of over-heated blood.

So that to regard a hypothetical proposition as implying a reciprocal hypothetical proposition, and an **A** proposition as justifying its reciprocal, or simple converse, are errors of the same kind. Both are fallacies of the Consequent.

(iv) CONTRAPOSITION.—We may perform both obversion and conversion in the order in which we have considered them, that is, first obvert the original proposition, and then convert the obverse thus obtained. This double process is called *Contraposition*, or *Conversion by Contraposition*; or, less appropriately, *Conversion by Negation.*

We may then take a further step and obvert the contrapositive. We thus reach a proposition of the same quality as that from which we set out, but with the negative of the original predicate as subject, and the negative of the original subject as predicate.

Starting with

A, the obverse is **E**, which converts to **E** (contrapositive).
E, ,, ,, **A**, ,, ,, **I** (,,).
O, ,, ,, **I**, ,, ,, **I** (,,).
I, ,, ,, **O**, which cannot be converted (no contrapositive).

This shows that the process yields most satisfactory results

[1] *Sophistici Elenchi*, ch. 5.

when applied to **A** propositions, for then there is no diminution in the range of the reference. In the contraposition of **E** the universal character of the judgement is lost in the second step; and the second step in the contraposition of **O** is but an insignificant verbal change.

We will, then, work out the process for an **A** proposition, and leave the reader to reach the contrapositives of **E** and **O** propositions. We may take as examples 'All unjust laws are opposed' (**A**), 'No unjust laws are unopposed' (**E**), and 'Some surgeons are not ill-paid' (**O**).

Original proposition :	**A** :	All unjust laws are opposed;
which obverts to	**E** :	No unjust laws are unopposed;
which converts to	**E** :	No unopposed laws are unjust;

which is the contrapositive of the original proposition.

From such an example we see that contraposition of a proposition brings out an implication which does not lie on the surface. This often appears yet more clearly when we examine the contrapositive in its equivalent obverted form. In the example before us this is 'All unopposed laws are just.' A writer might fail to see that in asserting the original proposition he was committing himself to this last. Yet it is only the same judgement stated in another way.

So we have found that one value of obversion and conversion is that they are steps in the more important process of contraposition.

(v) Implications of Hypothetical Propositions.—The formal implications of hypothetical propositions may be educed in a similar way, using the same forms as in Opposition.[1] Again, the contraposition of the universal affirmative is the most valuable. We will give it as an example, and leave the reader to work out the others for himself.

[1] See p. 75.

First, symbolically—

Original proposition:	If **X** is true, then always **Y** is true;
which obverts to	If **X** is true, then never **Y** is false;
which converts to	If **Y** is false, then never **X** is true;
or in obverted form	If **Y** is false, then always **X** is false.

Taking the same steps with a material example—

If taxation is just, every subject always pays a fair share;
If taxation is just, a subject never pays an unfair share;
If any subject ever pays an unfair share, the taxation is not just;
If any subject ever pays an unfair share, the taxation is unjust.

6. Rhetorical Equivalents.

(i) Varieties of Expression.—The processes we have considered are the formal modes in which we find what other propositions of the four typical forms can be educed from each of them, attention being concentrated on the denotation of their terms. It was pointed out at the beginning of the last chapter that the reduction of the multitudinous statements of ordinary speech to these typical forms is a mere matter of convenience for purposes of formal inference. Logic makes no such absurd claim as that in accurate thinking people should be limited to such bare forms of expression. Nor does it assert that the formal modes of educing implications are the only ones we actually and legitimately use.

A juagement can always be expressed in a variety of ways. We can say 'The curfew tolls the knell of parting day,' or 'The passing of the day is marked by the tolling of the curfew,' or 'The curfew is heard at the close of day,' or 'The day does not pass without the tolling of the curfew,' or 'Until the curfew is rung the day has not closed'; and, doubtless, the reader can find other equivalent forms.

If we would substitute one such form for another we must make sure that they make exactly the same assertion. Their

differences of form carry differences of emphasis and of standpoint, so that one would naturally occur in one train of thought, another in another. Each would most appropriately answer a different question. 'What is the meaning of the curfew?' 'What shows when day is ended?' etc. But each expresses the same fact—that a bell is rung as a sign that the day has reached sunset.

When statements are equivalent it is legitimate to substitute one for the other. The danger to be guarded against is similar to that of falling into the fallacy of the Consequent—the unjustified extension of the assertion made by the original proposition. The rule of eduction must be obeyed: whatever the change of wording there must be no increase of the range of application.

(ii) Reciprocal Propositions.—When we are limited to the normal forms of proposition used in formal inference we have found that conversion is a process of limited application. We can get no formal converse of a particular negative proposition, and when we convert a universal affirmative proposition we have to sacrifice its universal character. Only in the case of universal negative and particular affirmative propositions does formal conversion yield a true reciprocal—that is, a proposition exactly equivalent to the original one—and then, especially with particular affirmative propositions, the process is a trivial verbal change.

But when we free ourselves from the restrictions of these artificial forms we can find, for all the common statements of life, reciprocals which are really different expressions of the judgement. Instead of saying 'All logicians are critical' we may say 'Critical power characterises all logicians'; for 'Some critics are confused thinkers' we may substitute 'Confused thinking marks certain critics'; for 'Some clear thinkers have not studied logic' we may write 'The study of logic is not the only way to become a clear thinker'; we may replace 'No clear thinker could make such an ele-

mentary mistake' by 'The making of such an elementary mistake is inconsistent with clear thinking.'

Such variations show differences in line of approach: in the first of each pair of examples the person, and in the second the attribute, is emphasised. But in each pair the same judgement is expressed in two equivalent forms.

Similarly, in asserting an action we may bring into prominence either the agent or the object. 'Johnny threw a stone and broke the window' and 'The window was broken by a stone thrown by Johnny' state the same fact from different standpoints. To say that in changing people's opinions the pen is mightier than the sword is to affirm that for that purpose the sword avails less than the pen. 'The Chantry Acts led to the closing of a great many schools' and 'The closing of many schools was due to the Chantry Acts' are alternative forms of one assertion about the Acts of 1545 and 1548.

Even predications made with intransitive verbs may be made in reciprocal forms. 'Queen Anne died in 1714,' 'Death overtook Queen Anne in 1714,' 'In 1714 occurred the death of Queen Anne,' are alternative ways of stating the same fact.

(iii) Comparison with Formal Eductions.—Such differences give richness to language. Which form of statement we use is determined by the stream of thought in which it occurs. All that has gone before leads up to it, and all that follows springs from it. The whole is directed towards a definite purpose, and has set out from a specific starting-point. All this determines the words in which each successive phase of the thought clothes itself.

Clear apprehension of meaning, power of precise and varied speech, and a critical attitude towards verbal expression, are at once the determinants of such changes, and the safeguards against fallacy in evolving one form from another. These are matters of personal capacity, not of a general

method of procedure. They belong to the sphere of grammar and rhetoric, for they are questions of the correct and effective use of language.

Logic accepts as valid all evolutions of implication in which the rule of eduction is not violated. Before any of them can be used in formal processes they have to be reduced to one of the normal forms. But it is only for use in such processes that such reduction is needed. The formal eductions are simply the determination of which of those forms can be legitimately substituted for another in formal reasoning. They must not be regarded as artificial limitations placed by logic either on thought or on expression.

CHAPTER VI.

METHODICAL THINKING.

1. Characteristics of Methodical Thinking.

(i) Examples.—Let us consider the following passages—

(*a*) "'She always was clever,' said poor Mrs. Nickleby, brightening up, 'always, from a baby. I recollect when she was only two years and a half old, that a gentleman who used to visit very much at our house—Mr. Watkins, you know, Kate, my dear, that your poor papa went bail for, who afterwards ran away to the United States, and sent us a pair of snow shoes, with such an affectionate letter that it made your poor dear father cry for a week. You remember the letter? In which he said that he was very sorry he couldn't repay the fifty pounds just then, because his capital was all out at interest, and he was very busy making his fortune, but that he didn't forget you were his god-daughter, and he should take it very unkind if we didn't buy you a silver coral and put it down to his old account? Dear me, yes, my dear, how stupid you are! and spoke so affectionately of the old port wine that he used to drink a bottle and a half of every time he came. You must remember, Kate?'"[1]

(*b*) "I left my profession not because I had any fault to find with it, but because I would not be a scandal to the order. You know yourself that I was forced into it by interested guardians. My constitution was too weak to bear your rule. I had a passion for literature. I knew that I

[1] Dickens: *Nicholas Nickleby*, ch. 18.

could be happy and useful as a man of letters. But to break the vow was held a crime, and I endeavoured to bear my misery. My profession was a mistake. You will say that there was the year of probation, and that I might have known my own mind. What can a boy of seventeen brought up on books know of his mind? I was released. I was left to my own will to choose such form of life as would suit me, and I was lucky enough to find friends who saved me from falling into mischief."[1]

(*c*) In the fifteenth century "it is probable . . . that Latin, such as it was, was not unfamiliar to all. The bailiff's accounts are invariably written in it. That the monks and academics should universally employ Latin was intelligible enough; but it is surely unreasonable to conclude that bailiff and lord, master and servant, should have agreed to record transactions of vital importance and minute detail in a tongue which neither of them understood, or either was ignorant of. The likelihood that Latin was generally understood is further enforced by the frequency of political songs in Latin or in a maccaroni of Latin and English. Nor were these bailiffs men in any superior position. The bailiffs of Cuxham, father and son, from the days when Merton College became possessed of the estate to the time when the whole family perished in the plague and their chattels became the property of the lords of the manor, were serfs, and so described."[2]

(*d*) "Is it proper to man to act for an end?

(*R*) Of the actions done by man, those alone are properly called *human*, which are proper to man as man. Now man differs from irrational creatures in this, that he is master of his own acts. Wherefore those acts alone are properly called *human*, whereof man is master. But man is master of his

[1] Erasmus: *Letter to Fr. Servatius* (trans. by Froude: *Erasmus* Lect. 9).

[2] Thorold Rogers: *Six Centuries of Work and Wages*, ch. 6.

own acts by reason and will: hence free-will is said to be *a function of will and reason.* Those actions, therefore, are properly called *human,* which proceed from a deliberate will. Any other actions attributable to man may indeed be styled *actions of man,* but not properly *human actions,* since they are not of man as he is man. Now it is clear that all the actions that proceed from any power are caused by that power acting in reference to its object. But the object of the will is some end in the shape of good. Therefore all human actions must be for an end." [1]

(ii) PURPOSE AND ARRANGEMENT.—Even on a cursory reading the first is seen to differ from all the others in the absence of rational sequence. It wanders from the theme, and it is only after more digressions that the point is reached that Mr. Watkins said "that you were one of the most astonishing children he ever saw." Nor is the rambling ended, for this leads to the further reminiscence "I know it was he who said so, because I recollect, as well as if it was only yesterday, his borrowing twenty pounds of her poor dear papa the very moment afterwards." Everything that Mrs. Nickleby wished to say is contained in one sentence; the rest is entirely beside the mark.

In contrast with this, the thought in each of the other passages is worked out methodically. Every new statement adds support to the main thesis, and the effect of the whole is cumulative. These are all examples of orderly thinking, while the first is a specimen of desultory, unordered thought. It may be a caricature, but we need only recall some of our recent conversations and reveries to be convinced that at most it exaggerates features by no means uncommon.

It is seen, then, that the first characteristics of methodical thought are that it has a definite aim or purpose, and that it

[1] St. Thomas Aquinas: *Summa Theologica*, II. i., qn. i., art. 1 (trans. by Fr. J. Rickaby: *Aquinas Ethicus*, vol. i., pp. 1-2).

works out that purpose in an orderly sequence of ideas. There is no wandering from the way, each fresh step follows from that which precedes it and leads on to that which follows it. Nothing is omitted which is necessary to establish the conclusion, but, at the same time, nothing is superfluous.

We next note that in the first two passages a position is established by evidence drawn by memory from personal experience. Mrs. Nickleby wishes to bring forward testimony to the exceptional nature of her daughter; Erasmus to justify his quitting the monastic life. In the other two passages the argument is impersonal. The evidence for the general knowledge of Latin in the fifteenth century is that of pertinent historical facts; that for the purposive character of human action is found in certain general considerations. But, whatever the process by which a conclusion is established, it is successful and convincing in the degree in which it is set out in an orderly manner.

Methodical thought, then, proceeds straight to its goal, it marshals the evidence in the most effective way, so that the idea grows continuously clearer, and the assurance of its truth becomes stronger. It is skilful thinking, while such meanderings as those of Mrs. Nickleby are thoroughly unskilful, for there is so much that is disconnected with the aim that the sense is lost in a maze of words.

The two are contrasted in the same way as are skilful and unskilful performance of some bodily activity. For example, the skilled pianist strikes only the proper notes, to each he gives the correct stress and duration, and he gathers them into significant phrases, so that the meaning of the whole passage is conveyed to his hearers. But the unskilled player blunders along, striking many wrong notes and laying waste both time and rhythm, reducing the whole to a confused jumble of sounds. Or, the skilled bicycle-rider goes smoothly on his way, making in a straight line for his destination, while the tyro wobbles about, first to one side of the road,

then to the other, lucky indeed if he reaches his destination without a fall. Every one of the erratic movements both of the pianist and of the bicycle-rider results from a superfluous and unordered use of muscular force. Just so, Mrs. Nickleby's uncontrolled reminiscences are a worse than useless expenditure of both time and mental energy.

(iii) DESCRIPTIONS.—So it is throughout. For example, if a description is to succeed in raising a clear picture in the mind of the hearer or reader it must be so arranged that it first gives a general idea of the whole, and then makes that idea distinct by setting forth the details in such an order that each easily fits into its place. Take as an instance this description by Sir Thomas More—

"The island of Utopia containeth in breadth in the middle part of it (for there it is broadest) two hundred miles. Which breadth continueth through the most part of the land, saving that by little and little it cometh in, and waxeth narrower towards both the ends. Which fetching about a curcuit or compass of five hundred miles, do fashion the whole island like to the new moon. Between these two corners the sea runneth in, dividing them asunder by the distance of eleven miles or thereabouts, and there surmounteth into a large and wide sea, which by reason that the land on every side compasseth it about, and sheltereth it from the winds, is not rough, nor mounteth not with great waves, but almost floweth quietly, not much unlike a great standing pool; and maketh well-nigh all the space within the belly of the land in manner of a haven; and to the great commodity of the inhabitants receiveth in ships towards every part of the land. The forefronts or frontiers of the two corners, what with fords and shelves, and what with rocks, be very jeopardous and dangerous. In the middle distance between them both standeth up above the water a great rock, which therefore is nothing perilous because it is in sight. Upon the top of this rock is a fair and a strong tower builded,

which they hold with a garrison of men. Other rocks there be lying hid under the water, which therefore be dangerous. The channels be known only to themselves. . . . The outside or utter circuit of the land is also full of havens, but the landing is so surely fenced, what by nature, and what by workmanship of man's hand, that a few defenders may drive back many armies."[1]

Here there is no confusion, each new particular falls into its place, and we find the picture steadily growing in definiteness and precision. But were we simply given the particulars, the attempt to weld them together into some coherent whole would be continually balked. We should have to jump from point to point, uncertain what sort of a product we are to form, and feeling confusion growing ever more confounded. Such is the result of narrations constructed on the Nickleby model. That a description be enlightening it is not only needed that the narrator have a clear mental picture of what he wishes to describe, but that he have the power of thinking it as a whole composed of parts which draw their meaning from their position in that whole. This holds whether the description be of a place or of an event.

(iv) System.—Such a clear conception presents the matter as a system. For by system is meant this mutual inter-relation of whole and parts. Modern thought conceives the whole universe as systematic, and, indeed, unity is implied in the very name. It is, of course, a system of infinite complexity, in which subordinate systems of all sizes and forms of structure are woven together in every conceivable variety of way. It is systematic throughout, in its parts as in the whole. The smallest insect or pebble is a system in itself, a system which is an infinitesimal part not of one but of many wider systems, each of which in its turn is a constituent of many yet more far-reaching systems, and so on indefinitely. This

[1] *Utopia*, bk. ii. (Ralph Robinson's Translation).

is the assumption on which thought attacks the riddle of the universe, and the success which has attended it strengthens our faith in its truth.

The attack must be made piecemeal. It can be begun any where, for in a system every element leads by its relations to every other element, and as we have said, every partial system is itself an element, immediately or mediately, in the great total system of the universe. All the great 'branches of science,' whether dealing with the nature of man or with that of the material world, are wide-reaching systems by which thought tries to understand experience.

But it is not only in the advance of exact, or 'scientific,' knowledge that we think by the aid of systems. Ask me what I was doing on the 26th of October, 1911. My memory is a blank. Yet I can find an answer. I first ascertain from an old almanac that the date fell on a Thursday. I know what was my routine of life at the time, and I feel confident that I should remember any deviation from that routine. So I conclude that on that day I gave lectures at certain hours on certain subjects. The discovery of the day of the week gave me the point at which thought could enter the time-system of my life, and that system gave me the relation between the day and the occupation.

Similarly, if I am going on a tour I plan out my journey through study of the system of space relations between the places I am proposing to visit, as given on a map, and that of time relations as set forth in a railway time-table. But my plan is a new system, containing relations both of space and of time, and not found either on the map or in the time-table. It is constructed by my own thought, under the guidance of my own purpose. Once it has served its turn it can be discarded and forgotten.

At the same time, it will be well for me if it is not an imaginary construction. If my thought has connected towns by lines of railway which do not exist in the world of fact, or

if it has arranged trains at hours at which they do not run —as, for example, by confusing 4 p.m. with 4 a.m.—then in my journeyings I shall discover the weakness of the system of thought which they are trying to carry out. I do not suspect the system of reality of having suddenly changed; railways to have vanished or trains to have altered without warning the hour of their running; but I blame my own carelessness in not making sure of the facts on which I constructed the system in my thought.

(v) Assured Starting-point.—This leads us to see that, no matter how carefully our thought may work within a system of its own construction, yet, unless that system be a correct representation of that piece of real experience, it is not likely to reach the truth. No doubt we sometimes stumble on the truth by accident; but it is equally certain that no sane person will trust to doing this in any matter of even small importance.

Thus we see how necessary it is that a system of thought should have a solid foundation. We build the system, and if its foundation be laid in a fog or in a mirage it will prove but a 'castle in Spain.' So, another essential of methodical thought is that it should make sure that what it starts from is true.

Evidently, any particular system is either in the making, or is one already made which thought is revising, probably with a view, as in the passage from St. Thomas Aquinas or the description from Sir Thomas More, of setting it forth to others. The safest starting-point for a new system is the exact knowledge of facts. If we set out from general assumptions, then if those assumptions be disproved the whole edifice must fall into ruins; or, if they be denied the system of thought built on them is rejected, and our exposition seems to the reader to be but a beating of the air.

Now, a general judgement may have one of two origins. Either it is an axiom, of which the truth is self-evident and

so needs no proof, or it is a generalisation from experience. Probably it reached us from others and has received some confirmation in our own lives. But it is easy to suppose that what we have often heard, and so have come to believe through the mere force of custom, must be true. Many people are unable to conceive that their opinions can be open to honest doubt. Other folk's divergent opinions are plainly prejudices: their own are well-grounded convictions.

Yet, of the origin of many—perhaps most—of these assumed beliefs they can give no account. They have received them from others as part of the intellectual currency of their social circle; they have never tested whether they be pure metal or false alloy.

There is good in this steady holding to the traditions of the past, which, after all, do express 'the wisdom of our ancestors.' It is well not to be carried away by every new fashion of thought. But this caution should not be allowed to degenerate into obstinacy. A new doctrine should be required to produce its credentials, but those credentials should be examined, and when there is conflict with a preconceived opinion that also should be required to show a reason why it should be believed. That is the really scientific attitude of mind.

If, on the other hand, we start from facts, we must be careful lest we mistake what the facts are, choose the wrong facts, or omit facts which are pertinent to the inquiry but not altogether consistent with those we have selected. Otherwise, the general relation we support by this mistaken or defective evidence will be little likely to be a true one. We select the facts under the influence of some supposition we have made, or of some question we wish to answer. In each case we are liable to be influenced by our feelings. We cherish the supposition because it is ours, we favour one answer to the question rather than another. So we are tempted to ignore or to explain away all facts which do not

fall in with our preconceived ideas. Then the system of thought we construct will not agree with the system of reality. As well assure ourselves that a train, stated in the time-table to leave a junction five minutes before the train we are travelling by arrives there, will be late, and sure to leave it five minutes after. That may comfort us at the moment, but is likely to lead to disappointment in the end.

(vi) Principles of Methodical Thinking.—So we may sum up our discussion, and say that the principles of orderly or methodical thinking are—

(1) *Our thought should be purposive; that is, have a definite object.*

(2) *We should make sure that our thought begins with what is true and pertinent.*

(3) *Our thought should advance by related steps.*

(4) *We should endeavour to make our system complete.*

2. **Inference.**

(i) Nature of Inference.—We may, then, work out the constitution of a system either by finding the results of some statement of general relation, or by seeking for a relation which will bind together facts we have collected. In each case thought reaches a conclusion which is justified by the premises from which it starts, but is not contained in them. When we first work out the system these results may be new to us, or at least we see them under the new aspect of constituent elements in this new system. That is inference.

But a process of thought does not cease to be inferential when it has become familiar to you or to me. The essence of inference is that it unfolds the nature of a system by setting forth the consequences the mind is impelled to accept because it accepts something else. Euclid's demonstration of the equality of the angles at the base of an isosceles triangle is a piece of rigid inference; it connects by necessary relations the equality of angles at the base with that of the

sides. And this characteristic of the system is quite independent of whether it is known by this or that mind.

Any piece of inference, then, may have one of two aims, which are distinct though related. It may seek to learn a relation, or knowing the relation it may seek to work out its consequences. The former is called *inductive*, the latter *deductive*, inference.

(ii) Development of Inference.—In actual life we are constantly reasoning both inductively and deductively, and passing from one way to the other. As has been pointed out in an earlier chapter, we have a natural tendency to generalise our experiences, and we find at times that such generalisations are premature and need revision.[1] This discovery is made when we apply deductively the general beliefs we have based on our experience, or which we have accepted on the testimony of those about us, to some new case, which we find they will not fit. Thus we are led to suspect that what we have taken for truths are not truths at all.

So grows up the natural logic of common sense, of which scientific logic is the perfected form. To make our thought scientific—that is, exact—is not to introduce into it some new method, but only to use more correctly the methods which we have used from infancy, for they are the natural forms in which our mental life expresses itself.

It is all a continuous process of making explicit, or clear to ourselves, what is at first implicit, that is, operative without receiving attention. Inference is implicitly present in all our thought, but we can examine and test it only when we make it explicit, that is, set out before ourselves the evidence on which our conclusions are based.

That I am pleased or pained I know immediately. There is no evidence possible beyond the pleasure or the pain itself. So, no inference is involved. But directly we pass beyond

[1] See p. 43.

feeling to even the most elementary form of thought, inference comes in, and its presence is signalised by the possibility of error.

I recognise a friend, and at first sight I suppose that this is as immediate an experience as is a feeling of pain. But sometimes I find that my 'recognition' is a mistake. I had taken the person I saw for my friend because of his appearance. That is to say I had inferred 'That person has such an appearance; my friend has that kind of appearance; therefore, that is my friend.' The whole of this is implicit: we do not put it into words even to ourselves. But when we find we are in error we often do note the points of difference which cause us to reverse our judgement.

The first supposition of identity may even lead us to distort what is actually visible to us. A short time ago I was taking a walk with a friend, and we had been regretting that the old wind-mills which used to be such picturesque features of the landscape were so generally falling into decay. Suddenly, pointing to a hill about four miles distant, my friend exclaimed "There is one on the top of that hill." I looked and said "Oh no! that's the tower of Fairlight church"—a square tower with a small turret rising from one corner. "Oh! but it must be a mill, for I can see the sails going round." Yet, less than an hour later, when we were much nearer, he was convinced that it was the church tower after all. Now, we had both seen the same object with our eyes. But with his mind he had seen it as a wind-mill, that idea being just then prominent with him: I had seen it as a church tower because I knew the district.

This seeing with the mind is interpretation of the present experience by bringing it under some idea derived from the past. This is of the nature of inference. If the idea be a wrong one we misinterpret, so that the mind perceives something different from that which the senses receive; we put the experience into the wrong system.

To take a different case. The child rejects the powder because he implicitly infers that future experiences in which it plays a part will resemble those of the past. 'The powder was nasty yesterday; this seems to be a similar powder; therefore it will be nasty to-day.' That sort of inference we make continually in daily life. Past experiences not only give a meaning to present experiences, but a ground for expectation of future experiences. It is all implicit. Again, it is when the expectation is not fulfilled that we make the grounds on which it was based explicit to our minds.

So we see that explicit inference, or conscious thinking, is prompted by the failures of implicit inference. We set ourselves to reason consciously when we find we have made a mistake.

(iii) COMPARISON OF INDUCTION AND DEDUCTION.—Let us now refer again to the examples with which we started. Each of the first three brings forward facts and, except the first, each finds a relation which unites them in a system. They are inductive.

Each fact adduced by Erasmus is related to all the others and adds force to the conclusion that he was justified in leaving the cloister for the world. Together they support that plea, and their common relation to it shows them to be consistent and co-operative.

Similarly, the facts that bailiffs' accounts and popular songs were written in Latin fall into the system of thought of which the bond is the assumption that Latin was understood by many people even of the poorer classes. The outcome is a conception of the intellectual life of England in the fifteenth century which would lead to the search for such further evidence as the adequate provision of schools and their frequentation by boys of all classes. So the system would grow richer and fuller in its contents. Into that we need not enter, but it illustrates how the weaving of facts

into a system, by gathering them together as instances of a general idea, leads on to a search for cognate facts.

In the passage from Aquinas the inference is deductive, and proceeds in the opposite direction. From the general positions that mastery over his acts is what is specifically human in man, and that reason and will are characteristic of this mastery, it follows that the exercise of reason and will is specifically human; then by combining this with the position that reason and will imply purpose the conclusion is reached that human acts are purposive. The outcome is to make explicit what is implied in the conception of human nature.

We see then, that induction is the building up of systems of thought in harmony with the relations of the real world. It starts from a problem offered by facts which challenge explanation, because their place in the scheme of things is not apparent. Its object is to solve this problem, and this it can do only by binding the facts into a systematic whole in which each appears as a co-operative element. But the solution must stand the test of all further pertinent experience; for the one aim of inductive thought is to attain truth.

On the other hand, deductive thought assumes the general nature of a system to be known. It takes as its premises, or starting-point, the general relations of the system, and aims at increasing its fullness by showing the particular cases which are bound together by those relations. It cannot test the truth of these assumed relations; that can be done only by induction; for its own purposes it postulates them as true. The test of the validity of its inferences is that they involve no inconsistency within themselves. If the conclusion of a consistent deductive inference as to fact is found to be at variance with fact, it is certain that the premises from which it is drawn are mistaken. So deduction is operative in testing inductions.

But the system within which thought works need not be one

representative of the world of fact. There are also open to our thoughts various worlds built up by the imagination; such as the worlds of fiction and of mythology. Within them thought not only can, but does, work on the same lines as when it is dealing with actual things and events. Of course its conclusions hold only in the world in which the premises are found.

The danger of confusing the imaginary with the actual is especially great in deduction; for, as has been said, we are all too prone to take our prejudices, which are often imagined as truths, for real truths. But induction can deal with the imaginary facts of Hamlet's acts and thoughts and from them form an idea of his character which will bind them all into a harmonious system, on the same lines as it can deal with other recorded facts or with facts open to observation. Indeed, the 'facts of history' are often fiction, and those of observation are fictional whenever a mistake is made about them.

The distinction between induction and deduction is not that the former deals only with the world of fact while the latter is indifferent to truth, but that each works out in its own way systems which are of worth to the degree to which they increase our understanding of the world to which they belong.

(iv) Analysis and Synthesis.—In each of our examples, the inductive reasoning found the general relation by analysis of the particular cases. The bailiffs' accounts contain many other elements to which attention could be directed besides that of the language in which they are written; *e.g.* the prices of wheat, cattle, and sheep, the wages of labourers, the conditions of tenure by which the tenants held their land. All these were put on one side, and attention was fixed on the language. So with the songs. Their political allusions, the consequent date of their composition and popularity, were ignored, and again the language was the only thing examined.

On what ground was such a selection made? Clearly on that of their bearing on the problem which was being investigated. Had another problem, such as wages, been taken up, another selection would have been made. The general relation was first conceived as a possibility. Then, with that as a clue, matters likely to furnish evidence for or against it were examined, and as the supporting evidence accumulated the probability that the supposition was a true one was strengthened, until it became practically a certainty.

So it is always. We never analyse facts at random, but always under the guidance of some question to be answered, or of some supposition to be tested. Thus, just as in judgement we at once analyse a given whole and hold together, or synthetise, the elements we find in it, so in inductive reasoning we both choose our examples and analyse them under the guidance of a purpose which is the synthetic bond that keeps our investigation from being mere random inquiry.

That induction is both analytic and synthetic is seen in that among the relations it establishes are those of similarity and dissimilarity, with the result that we revise the rough classifications and definitions involved in learning a language. To state the result of an examination into the nature of things is to define them, and at the same time to separate them in thought from things of a different nature and to class them with those of a like nature. So classification and definition are two sides of one process. The one gives an orderly exposition of the denotation, the other of the connotation, of our terms. By this means when a relation is established by the analysis of a particular example it is generalised as true of the class, and the resulting system is freed from all limitations of space and time.

On the other hand, in deduction the most obvious aspect of the thought is the inclusion of particular cases in a system by bringing them under a general rule. But this can be done

only when an identical element is found in both rule and cases. In the example from Aquinas this bond is reason and will, which imply both mastery over acts and the presence of purpose. But to find an element in an idea which contains it we must analyse the idea. So the synthesis of deduction is effected through analysis, even as the analysis of induction is determined by the synthetic bond of the purpose of constructing a system.

It appears, then, that, though on their face induction and deduction are opposed forms of reasoning, yet each is both analytic and synthetic. They are, therefore, not antagonistic but complementary. Each is the working out of a system.

3. **Empirical Laws and Axioms.**

(i) NATURE OF EMPIRICAL LAWS.—General relations established by induction are known as *Empirical Laws*; that is, relations learned by analysis of experience, and supported by the evidence of experience. If the facts on which such a law is based have been wrongly or inadequately selected or interpreted, the resulting generalisation will need revision and modification, even if it be not altogether overthrown. By induction men are ever grasping relations in nature hitherto unknown, or learning more perfectly the scope of those already discovered. They name their statements of these discoveries 'Laws of Nature'; but it must be remembered that this use of the word 'law' implies nothing further than a uniform relation between facts, so far as it is known.

All empirical laws, then, are liable to revision should experience furnish evidence that it is demanded by the facts of the case. It is obvious that the wider the scope of the generalisation the less likely is it to be overthrown. Indeed, in some cases overthrow has become incredible. Every day gives us a vast mass of evidence of the truth of that relation between portions of matter which we call gravitation, and never any evidence repugnant to it. So we should reject without

hesitation any assertion of the existence of material bodies between which it did not hold. Yet that theory has been a part of human knowledge but for little more than two centuries. The facts were there, and as separate events and happenings were known to everyone, but Newton was the first to gather them into a system wide as the material universe itself.

(ii) Testing.—A rigid deduction from an empirical generalisation can be no stronger than the weakest link in the chain of inductive evidence which led to its formulation. It is by further comparison with fact of conclusions reached deductively from empirical laws that those laws are tested, and as a result of the testing, established as practically certain, modified, or rejected.

So verification by ordered appeal to experience to confirm the results deduced from assuming the empirical generalisation to be true is the final step of induction. But it is not a step which is taken once for all. Rather it is one which is continually being taken, now with this piece of pertinent experience, now with that. Induction, then, is not mere generalisation from experience; it is the whole process by which generalisations are established as true.

(iii) Axioms.—An axiom, on the other hand, is not liable to be overthrown by fresh evidence, for its evidence is in itself. That the whole is greater than its part cannot be proved. Thought cannot get below it to any more elementary truth on which it rests. So with all axioms. We cannot even think their terms in a different relation. It follows that a rigid deductive inference from an axiom reaches an incontrovertible conclusion.

We may, however, take for axioms statements that are not axioms; that is, which have not this ultimate character. If in any case the conclusions we draw from what we have assumed to be an axiom do not agree with the facts of experience the supposed axiom must be rejected. In the

light of the new experience we then see that we have taken a mere prejudice of thought for a self-evident truth, for that which can be controverted by evidence needs evidence outside itself to support it, and cannot, therefore, be axiomatic.

So in every case a universal judgement must agree with facts or be rejected.

CHAPTER VII.

FALLACIES INCIDENT TO METHOD.

1. Nature of Fallacy.

(i) Definition of Fallacy.—Our last chapter has shown that methodical and accurate thinking is a work of skill, which is acquired only by much careful practice. At every step we may go wrong. So we rightly expect that a logical examination of thought will point out the kind of mistakes against which we need to be on our guard, both in working out a line of thought for ourselves and in following one offered to us by another.

The most dangerous errors are those which are hidden under a specious show of correctness. Some rule of exact thinking is violated, but the violation is covert; on the surface the inference appears valid. Such an error is rightly named a *Fallacy*. The loose use of that term to denote any kind of mistake should be avoided.

Ambiguities of language, involving confusion of thought, such as we considered in the second chapter, offer many pitfalls to the unwary. They make it easy to fall into fallacy in our own thought, and often render it difficult to detect faults in the reasonings of others, whether those faults be committed intentionally or unintentionally.

(ii) Fallacy and Sophism.—When fallacious arguments are deliberately offered for the acceptance of another they are properly termed *Sophisms*, a name derived from that of the old Greek teachers known as Sophists, whom both Plato

and Aristotle accused of cultivating the art of winning verbal victories over an opponent, without regard to truth or honesty. Such sophistical reasonings were common features of dialectic disputation, and it was in this relation that Aristotle first analysed and classified them.

But it is by no means impossible to use sophistical reasonings in that private dialectic in which we weigh arguments for and against a proposed course of action, or a suggested conclusion. On the contrary, we are all prone to do so when our desires are in conflict with our principles. Then we need be very watchful over ourselves lest we drug our consciences with specious reasonings, which we are dimly conscious will not stand the test of candid and critical examination. At other times we fall into fallacy honestly, n that we do not recognise the defective character of our thought.

The distinction between simple fallacy and sophism is a psychological and ethical one, depending on whether or no the invalidity is conscious and deliberate. Logically there is no distinction, for the kind of violation of the principles of clear thinking is the same when it is, and when it is not, intentional.

(iii) Fallacy and Truth.—It should be noted that it does not follow from the presence of fallacy in an argument that the conclusion reached is itself false; but only that it is not proved by the reasons brought forward. A true position may be supported by bad reasoning. On the other hand, sound reasoning will reach a false conclusion if it is based on false assumptions. Our thought is satisfactory only when it is correct in the conclusions it reaches, in the grounds on which they are based, and in the connexions it makes between grounds and conclusions.

(iv) Fallacies of Method.—The fallacies incident to the methodical ordering of our thoughts on any topic cluster round the conception of the aim or purpose, the facts

or assumptions from which we set out, the passage from the latter to the former, and the completeness with which relevant matter is taken into consideration.

Nor are these mutually exclusive. The human mind is quite capable of making more than one mistake in the same piece of reasoning. Indeed, it is not uncommon for the commission of one fallacy to prepare the way for another. So it is often possible to refer a mistake to more than one class. Nevertheless, the kinds of sources of error may profitably be distinguished.

2. **Fallacies related to Purpose.**

Those of the first group are the most common and the most insidious. Any defect in the clearness with which we keep the actual question or problem in view affects the pertinence of the whole process of thinking. We will, therefore, first consider some typical cases of failure to keep to the point, though it is impossible to enumerate all the ways in which people both can and do waste their intellectual efforts by wandering from it.

IRRELEVANCE TO PURPOSE [IGNORATIO ELENCHI].

(*a*) *Changing the Issue.*—When the George the Fifth postage stamps were first issued there was general dissatisfaction with both their design and their material. In the House of Commons Mr. Bennett-Goldney asked the Postmaster General, among other questions, "whether he would arrange for the stamps to be printed on less tearable paper." To which Mr. Samuel replied: "The paper used is the same in the new as in the old stamps."[1] This mode of avoiding the giving a straight answer to an inconvenient question is one of the most common devices, not only of Ministers of the Crown, but of all who wish at once to appear to give satisfaction and to avoid doing so.

[1] *The Times*, June 29, 1911.

Similar to this is that favourite device of the sophist who cannot overthrow an opponent's real argument, of setting up in its place a rhetorical 'man of straw' and then proceeding to demolish the simulacrum with loud shouts of victory. Slight changes and omissions in statement are often enough, and the fewer they are the more likely is the sham refutation to be taken for a real one.

To take an example from the famous *Letters of Junius*: "A friend of JUNIUS desires it may be observed (in answer to *A Barrister at Law*) . . . JUNIUS did never say that Lord Mansfield had *destroyed* the liberty of the press. 'That his lordship has *laboured to destroy*—that his doctrine is an *attack* upon the liberty of the press . . .' are the propositions maintained by JUNIUS. His opponents never answer him in point, for they never meet him fairly upon his own ground."[1]

(*b*) *The Argumentum ad Hominem,* or *Attacking the Adversary instead of his Arguments.*—Of these fallacies we may instance—

(1) *The 'Tu Quoque' Retort.*—Mr. Lloyd George thus began a speech in Parliament on the National Insurance Bill: "The honourable gentleman who has just sat down talked a good deal about misrepresentation. It requires a good deal of courage for any gentleman belonging to the party opposite to speak about misrepresentation in connexion with this Bill."[2] Perhaps; but such a crude use of the 'You're another' [*Tu quoque*] style of repartee, so common in vulgar quarrels, is no answer to the charge made. A man does nothing to establish his own honesty by asserting that he who impugned it is not above reproach on the same count. Two blacks do not make a white.

A more subtle form of this spurious kind of refutation is addressed by JUNIUS to Sir William Draper: "You assure

[1] Letter 63. [2] *The Times*, Dec. 7, 1911.

me that my logic is puerile and tinsel, that it carries not the least weight or conviction, that my premises are false and my conclusions absurd. If this be a just description of me, how is it possible for such a writer to disturb your peace of mind, or to injure a character so well established as yours?"[1] This neatly insinuates that Sir William's own mental powers were properly described by the very terms he had applied to those of his adversary.

(2) *Abuse of Opponent.*—In the first of the famous series of collected letters, JUNIUS speaks of "strong assertions without proof, declamation without argument, and violent censures without dignity or moderation." Of each of these forms of invective he was himself a past master. His method was pretty accurately described by Sir William Draper: "Suspicion is the foul weapon with which you make all your chief attacks; with that you stab."[2]

To the Duke of Grafton JUNIUS wrote: "The character of the reputed ancestors of some men has made it possible for their descendants to be vicious in the extreme without being degenerate. Those of your Grace, for instance, left no distressing examples of virtue, even to their legitimate posterity, and you may look back with pleasure to an illustrious pedigree, in which heraldry has not left a single good quality upon record to insult or upbraid you."[3] To Lord Mansfield: "Our language has no term of reproach, the mind has no idea of detestation, which has not already been happily applied to you, and exhausted."[4]

Mr. Horne, another of his victims, wrote to him: "You brought a positive charge against me of corruption. I denied the charge, and called for your proofs. You replied with abuse and re-asserted your charge. I called again for proofs. You reply again with abuse only, and drop your accusation."[5]

[1] Let. 25. [2] Let. 26. [3] Let. 12. [4] Let. 41. [5] Let. 56.

This quite well describes a method of ignoring the point at issue which has always been common, and will continue to be so while the inflaming of men's passions is more effective than are appeals to their reason. "Close and relevant arguments have very little hold on the passions, and serve rather to quell than to inflame them; while in personalities, there is always something stimulant, whether on the part of him who praises or him who blames."[1]

True, less robust forms of abuse are now in fashion, but the fallacious character of such retorts is not thereby diminished. Public measures are still opposed on such grounds as that the proposers introduce them with a bad motive, have some evil further design, are themselves examples of anything but probity, or associate with those whose character is open to suspicion. All such considerations are utterly foreign to the question whether a measure is good or bad. That can only be decided by an estimate of its probable consequences.

Other forms of appeal to the emotions, more insidious than abuse, are those that masquerade as appeals to reason. From the point of view of clear and pertinent thinking, all rhetoric is suspect.

To hold an opponent up to opprobrium or to ridicule is no proof that he is wrong, and you are right. Rather, indeed, it suggests that you are conscious that the opposite is the case, and wish to hinder others from perceiving it. So with the extreme form of persecution for opinion. That shows, indeed, that the persecutors dislike the opinions for the holding of which they impose penalties, but it is no evidence that those opinions are either false or wicked.

(3) *Charge of Inconsistency.*—One special form of attacking the opponent instead of his arguments is to charge him with inconsistency. Thus JUNIUS: "You, Mr. Ellis, . . .

[1] Bentham: *Book of Fallacies*, pp. 141-142.

are inconsistent with your own principles. You have hitherto maintained that the House of Commons are the sole judges of their own privileges, and that their declaration does, *ipso facto*, constitute the law of parliament; yet now you confess that parliaments are fallible, and that their resolutions may be illegal, consequently that their resolutions *do not* constitute the law of parliament."[1]

That a man has changed his opinion on a particular point is, in itself, no argument either for or against either of the views he has supported, though it may be pertinent to the question of the stability of his character.

The case is different when inconsistent arguments are simultaneously used by the same person. Then they destroy each other, and to point this out virtually leaves the position without support.

Such a legitimate retort of inconsistency JUNIUS makes in a letter to Lord Mansfield: "In the two last trials, your charge to the jury began, as usual, with assuring them that they had nothing to do with the law,—that they were to find the bare fact, and not concern themselves about the legal inferences drawn from it, or the degree of the defendant's guilt. Thus far you were consistent with your former practice. But how will you account for the conclusion? You told the jury that, 'if, after all, they would take upon themselves to determine the law, *they might do it*, but they must be very sure that they determined according to law.' . . . In the first instance you deny the power absolutely. In the second, you admit the power, provided it be legally exercised."[2]

Nor is Mr. Horne's retort of inconsistency against JUNIUS illegitimate: "When JUNIUS is called upon to justify his accusation, he answers, 'he cannot descend to an altercation with me in the newspapers.' JUNIUS, who *exists* only

[1] Let. 48. [2] Let. 41.

in the newspapers, who acknowledges 'he has attacked my character' *there,* and 'thinks I have some right to an *explanation*'; yet this Junius 'cannot descend to an altercation in the newspapers!' and because he cannot descend to an altercation with me in the newspapers, he sends a letter of abuse by the printer, which he finishes with telling me—'I am at liberty to *publish* it.' This to be sure is a most excellent method to avoid an altercation in the newspapers!"[1]

(*c*) *Transferring the Onus of Proof.*—A fallacy of irrelevance is also committed when an attempt is made to throw the burden of proof on the wrong side.

Sir William Draper fell into this when he wrote: "Junius makes much and frequent use of interrogations; they are arms that may be easily turned against himself. I could, by malicious interrogation, disturb the peace of the most virtuous man in the kingdom; I could take the decalogue, and say to one man, Did you never steal? To the next, Did you never commit murder? And to Junius himself, who is putting my life and conduct to the rack, Did you never bear false witness against thy neighbour? Junius must easily see, that unless he affirms to the contrary in his real name, some people who may be as ignorant of him as I am, will be apt to suspect him of having deviated a little from the truth: therefore let Junius ask no more questions. You bite against a file; cease viper."[2]

The advocate of any particular change in social or political arrangements ought to show that it will be advantageous to the community. To call on those who disagree with his remedy for an ill which all may admit to prove that it will not work, or will work ill, is to ignore this fundamental requirement.

On the other hand, it is fallacious to conclude that the

[1] Let. 53. [2] Let. 6.

existence of real objections to a scheme should at once condemn it. This is an imperfect world, and against most human arrangements something can be justly urged. The pertinent question is whether the objections to a scheme are more, or less, weighty than the considerations in its favour.

(*d*) *Irrelevance in Private Thought.*—We may commit any of these fallacies in our own thought. We may give our prejudices or our feelings the decisive voice in matters that should be dealt with by cool reasoning. We may colour darkly the acts of one who has offended us, and so justify to ourselves a line of conduct towards him which, otherwise, we could not approve.

When we are seeking the solution of a problem, whether it be theoretical or practical, we are likely to err if we have not made sharply explicit to ourselves the exact nature of our difficulty. Then, when something, in some way related to it, becomes clear, we assume that we fully understand the matter, when in truth we have passed the real obstacle on one side.

This pitfall is inherent in illustrative concrete examples of abstract laws and truths. The example is understood in some of its many aspects, but it may be that those are irrelevant to the relation it was intended to illuminate, and that that remains in the shade. Many a boy thinks he understands an arithmetical process because he has followed a particular example worked out by his teacher, and only discovers his mistake when he attempts to solve a somewhat different problem by his own unaided efforts.

The safeguards against irrelevance are: to be quite clear as to what is the problem we are trying to solve; to keep the question we are to answer, whether to ourselves or to another, steadily before us; and to endeavour to see it in the light of truth, not distorted by our wishes and prepossessions.

3. **Fallacies related to Starting-point.**

(i) Begging the Question [Petitio Principii].

(*a*) *Question-begging Epithets.*—In a letter to *The Times* on Lord Lytton's 'Conciliation Bill' Sir Oliver Lodge wrote: "Surely the time has come for reasonable men of all parties to combine in removing an antiquated and artificial disability which debars certain taxpayers from taking their due share in representative government, no matter how otherwise qualified they may be, solely on the irrelevant ground of sex."[1]

But the very point in dispute is whether sex *is* irrelevant to the exercise of the franchise. And, though the disability is of old date and imposed by law, yet the words "antiquated and artificial" not only imply this, but suggest that it is indefensible in modern life, and opposed to some right inherent in human nature. Thus, by the use of these terms the question is begged. They assume as incontrovertible the very conclusion the writer aims at establishing.

To take another example. In 1770, Junius wrote: "One particular class of men are permitted to call themselves the King's friends, as if the body of the people were the King's enemies; or as if his Majesty looked for a resource or consolation, in the attachment of a few favourites, against the general contempt and detestation of his subjects."[2]

It is this suggestion of something more than the strict meaning of the words which makes the use of question-begging epithets so insidious a form of fallacy in our own thought, and one so often difficult to detect or to expose in the arguments of others.

Among terms, especially those that refer to men and their deeds, "there are some by which the object is presented singly, unaccompanied by any sentiment of approbation or disapprobation attached to it: as, *desire*, *labour*, *disposition*, *character*,

[1] July 2, 1911. [2] Let. 39.

habit, etc. With reference to the two sorts of appellatives which will come immediately to be mentioned, appellatives of this sort may be termed *neutral*.

There are others by means of which, in addition to the principal object, the idea of general approbation, as habitually attached to that object, is presented: as, *industry*, *honour*, *piety*, *generosity*, *gratitude*, etc. These are termed *eulogistic* or laudatory.

Others there are again, by means of which, in addition to the principal object, the idea of general disapprobation, as habitually attached to that object, is presented: as, *lust*, *avarice*, *luxury*, *covetousness*, *prodigality*, etc. These may be termed *dyslogistic* or vituperative. . . .

As to the mode of employing this fallacy, it neither requires nor so much as admits of being taught: a man falls into it but too naturally of himself; and the more naturally and freely, the less he finds himself under the restraint of any such sense as that of shame. The great difficulty is to unlearn it: in the case of this, as of so many other fallacies, by teaching it, the humble endeavour here is to unteach it.

In speaking of the *conduct*, the *behaviour*, the *intention*, the *motive*, the *disposition* of this or that man, if he be one who is indifferent to you, of whom you care not whether he be well or ill thought of, you employ the *neutral* term:—if a man whom, on the occasion and for the purpose in question, it is your object to recommend to favour, especially a man of your own party, you employ the *eulogistic* term:—if he be a man whom it is your object to consign to aversion or contempt, you employ the *dyslogistic* term.

To the proposition of which it is the leading term, every such eulogistic or dyslogistic appellative, secretly as it were, and in general insensibly, slips in another proposition of which that same leading term is the subject, and an assertion of approbation or disapprobation the predicate. The person, act, or thing in question is *or* deserves to be, or is *and*

deserves to be, an object of general approbation; or the person, act, or thing in question is or deserves to be, or is and deserves to be, an object of disapprobation.

The proposition thus asserted is commonly a proposition that requires to be proved."[1]

Illustrations from daily life are innumerable. A change is to those who advocate it, 'an improvement'; to those who oppose it, 'an innovation' or 'a revolution': what is 'liberty' to him who desires it is 'license' to him who disapproves of the proposed relaxation of rule: the 'strong government' of the ruler is apt to appear 'tyranny' to the subjects: the same material surroundings are 'necessaries of life' to those accustomed to enjoy them, 'gross luxury' to those who lack and envy them.

Names of parties and movements are frequently of a question-begging character. 'Progressives' suggests that the rival party is opposed to progress, and 'Moderates' that the other side consists of men who are unbalanced in their desire for change. 'Free Trade,' 'Fair Trade,' and 'Tariff Reform' are equally successful in surreptitious appeal to the feelings and in illegitimate suggestion.

(*b*) *Assuming the Conclusion.*—The question may be begged in other ways. The copiousness of our vocabulary makes it possible to assume as the starting-point of an argument a mere paraphrase of the conclusion to be established. We may be assured that a certain salve will reduce irritation, because it is an emollient. Or a conclusion may claim to be drawn from a correlative proposition stating the same fact; as that Brighton is south of London, because London is north of Brighton.

(*c*) *Circular Reasoning.*—When a proposition is assumed as the starting-point of one step of an argument, and drawn as a conclusion from another step, the reasoning is appropri-

[1] Bentham: *Book of Fallacies*, pp. 214-216.

ately styled 'circular.' When the two steps are set out side by side, the fallacy is made manifest, but when they are separated from each other by, perhaps, many pages, it is likely to escape the notice both of the writer and of the reader. In the eighteenth century Dr. Priestley in his *History of the Corruptions of Christianity* "argues that St. John's language is to be understood in the Unitarian sense, because the early Christians were Unitarians; and that the early Christians were Unitarians because St. John preaches Unitarianism."[1]

(*d*) *Supposed Axioms.*—One of the most fertile sources of error in the history of thought has been the assumption that certain general propositions, which involve the conclusions drawn, are axiomatic.

Says JUNIUS: "If it be really a part of our constitution, and not a mere *dictum* of the law, *that the King can do no wrong*, it is not the only instance, in the wisest of human institutions, where theory is at variance with practice. That the sovereign of this country is not amenable to any form of trial, known to the laws, is unquestionable. But exemption from punishment is a singular privilege annexed to the royal character, and no way excludes the possibility of deserving it. . . . A mistake in this matter proved fatal to Charles and his son."[2]

JUNIUS himself committed this fallacy when he wrote to the Duke of Bedford: "My Lord, we are too well acquainted with your pecuniary character, to think it possible that so many public sacrifices should have been made, without some private compensations."[3]

If anyone should argue that no trust should be reposed in M. Venizelos, because he is a native of Crete, and we have St. Paul's testimony that "The Cretans are always liars,"[4]

[1] Sir Leslie Stephen: *English Thought in the Eighteenth Century*, vol. i., p. 438.

[2] Preface to *Letters*. [3] Let. 23. [4] *Titus*, i. 12.

doubtless the argument would be scouted; though, had the Greek statesman taken the opposite line to that he has taken, it is not unlikely that some popular orator or newspaper would have regaled us with it.

A vast amount of reasoning has been fallacious because it has rested on supposed axioms, which experience has since proved false. No age has been free from them, and there is no branch of knowledge in which progress has not been hindered by them. Advance in knowledge of the material world was hampered for centuries by the belief that the ancients, especially Aristotle, had exhausted the field. An admirer of Aristotle "refused to look through a telescope, lest he should be convinced of the existence of Jupiter's moons."[1] The infallibility of the ancients was generally regarded as axiomatic.

The seventeenth and eighteenth centuries were especially prolific of supposed axioms regarding the nature of man. The divine right of kings led to acquiescence in much misgovernment, till the opposed divine right of rebellion appeared to some as equally axiomatic. Then followed civil troubles in England, and, a century later, revolution in France.

That all human impulses are evil was regarded as axiomatic by the Jansenists; that the same impulses are good was self-evident to Rousseau. The former attributed all actual good in man to the direct compulsion of divine grace; the latter saw in the corrupting influence of society the origin of all human faults.

Underlying these and similar assumptions was the supposed axiom that human nature in itself is colourless and formless, alike in every individual, so that men and women owe all their personal characteristics to influences external to them. To account for society it was assumed that, at some early time,

[1] Dr. C. Mercier: *Causation*, p. 205.

men entered into a 'social contract' to live together in peace, so that "the state rests upon the voluntary consent of mankind to trust the magistrate with powers necessary for the protection of their civil interests."[1]

Only while these assumptions were accepted as axiomatic, and so needing no proof, could they blind men's thought. As soon as each is seriously challenged it falls like a pack of cards. "The imaginary [social] contract has confessedly no place in history, and it is easy to show that it can have as little in philosophy."[2] But for a couple of centuries they misled and confused men's thoughts on subjects of the greatest interest and importance.

It would be hard to find a more striking illustration of the lengths to which the mania for putting forth ungrounded assumptions as axiomatic truths may go than Bishop Warburton's claim: "In a word, an established religion with a test-law is the universal voice of nature."[3]

In still later times the favourite assumptions have been of a materialistic character. The tendency is to assume that our senses are not only gateways of knowledge of the world about us, but determinants of the possibilities of existence. Looking back on the history of thought, we should do well to take to heart the words of Sir Oliver Lodge: "That the earth is a speck invisible from any one of the stars, that we are on a world which is but one among an innumerable multitude of others, ought to make us realise the utter triviality of any view of existence based upon familiarity with street and train and office, ought to give us some sense of proportion between everyday experience and ultimate reality."[4]

[1] Stephen: *Op. cit.*, vol. ii., p. 149.

[2] *Ibid.*, vol ii., p. 179.

[3] *Divine Legation*, quoted by Sir L. Stephen, *Op. cit.*, vol. ii., p. 166.

[4] Article on *The Reality of Continued Existence: The Daily Chronicle*, Nov. 14, 1916.

One of the greatest services of modern science to human thought has been to make men critical of assumptions. Unhappily, the assumptions of our own day are the last to which we think it necessary to apply the test to which we submit those of earlier thinkers.

(*e*) *Paradoxes.*—Some assumed axioms are contrary to current opinion. These are called paradoxes. "To maintain that all intelligences are equal (Helvetius), that arts corrupt manners (Rousseau), that property is theft, that anarchy is the true form of government (Proudhon), that the animals are our superiors (Montaigne): these are true paradoxes."[1]

But there are other seemingly paradoxical judgements which are really in harmony with the laws they seem at first sight to contradict; such as, a body heavier than air can fly through the air, an asymptote is a line which approaches nearer and nearer to a straight line, but never reaches it.

Roger Bacon's anticipations that "Instruments for navigation can be made which will do away with the necessity of rowers. And carriages can be constructed to move without animals to draw them, and with incredible velocity. Machines for flying can be made" were in the highest degree paradoxical to his thirteenth century contemporaries: to us they are the commonplaces of daily life.

A true paradox, then, is in opposition to a true law. Others may be inconsistent with men's experience hitherto, and yet become parts of the knowledge and life of the future.

(*f*) *Fallacious Use of Analogy.*—A favourite rhetorical mode of begging the question is to assume that a metaphor or simile has probative force.

An analogy is an identity in relation. Fallacy arises if this identity is extended to other relations.

Thus, colonies are the offspring of the 'mother' country. To infer from that a parental power in the latter to determine

[1] Cardinal Mercier: *Logique*, p. 245.

the action of the former is to beg the question as to all relations outside that of origin, to which alone the metaphor applies.

Similarly, a community is like an individual in being an organised whole, with distinct personality, aims, and duties. To infer that as years advance a community must enter into a period of senile decay, as is the fate of every individual, is to beg the question as to the nature of the life of society.

Equally fallacious are conclusions drawn from the nature of the terms of an analogy, for similar relations may hold between very different sets of terms. "If I say 'You are as slow as a slug,' 'You serve him as faithfully as a dog,' . . . my interlocutor would be apt to turn upon me with indignation and demand 'Do you say I am a slug?' 'Is thy servant a dog that he should do this great thing?'"[1] If any implication of the kind is intended, the question as to character is begged.

How easily the same relation may be expressed in different terms is evidenced by the facility with which an orator 'mixes' his metaphors in attempting to drive home his meaning. Mr. R. Blatchford had but one idea in his mind when he wrote in *The Weekly Dispatch* "In this insidious manipulation of the thin end of the Tory wedge do we not perceive the cloven hoof of the serpent casting its shadow before?"[2]

Equally fallacious is it to conclude that differences in many circumstances make it impossible for there to be a real likeness in relation. Melons differ very much in appearance. "They differ chiefly in their fruits, but also very much in foliage and mode of growth. Some melons are only as large as small plums, others weigh as much as sixty-six pounds. One variety has a scarlet fruit. Another is not more than an inch in diameter, but sometimes more than a yard in length, twisting about in all directions like a serpent. Some melons

[1] Dr. C. Mercier: *A New Logic*, p. 352. [2] *Punch*, vol. cl., p. 270.

are exactly like cucumbers; and an Algerian variety, when ripe, cracks and falls to pieces, just as occurs in a wild gourd."[1] But to assume that any one of these varieties is not a melon is to beg the question as to the possibility of identity of nature being combined with variety of appearance.

(*g*) *Begging the Question in Induction.*—When our thought starts from particular instances, or facts, we beg the question whenever we select such facts as favour the conclusion we wish to establish, ignore those that are opposed to it, or, if we cannot do that, distort them by exaggerating some of their aspects and neglecting others.

Many people used to believe that dreams were prophetic. They dwelt solely on those that had 'come true,' though the 'coming true' was often established only by allowing a very generous margin of discrepancy between dream and fact. Indeed, the rule of interpretation was not sure; for did it not also become proverbial that 'dreams go by contraries'?

'To meet a black cat is lucky, for many people have met black cats shortly before some good fortune has come to them.' Yes, doubtless. But how about the innumerable times when the cat is met and no happy result follows, or the smaller number when the good fortune has come but has omitted to send the sable messenger?

The overlooking of pertinent facts is often called the *Fallacy of Non-observation*; that of taking facts to be other than they are, the *Fallacy of Mal-observation*. This they are; the fallacy consists in taking a part for the whole, and so in giving a wrong interpretation to what is observed.

But we always collect facts with a purpose. We want to answer some question, or to solve some problem; and often we wish the answer or solution to harmonise with an idea we already have. Then, if we consciously choose and manipulate the facts to make them fit into our theory or support our

[1] Wallace: *Darwinism*, pp. 87-88.

wishes, we sophistically beg the question as to the truth of that favoured solution.

So, too, we beg it—though not sophistically—when, without adequate inquiry, we jump to the conclusion that we have taken account of all the pertinent facts, and that they are all as we assume them to be. This is similar to the assumption without examination of some general proposition which we illegitimately regard as axiomatic.

Into the difficulties of observation we shall not enter here. They will be discussed in the next chapter. Sufficient now is it to point out the need to be on our guard against all attempts to make facts fit into our theories, instead of moulding our theories into accord with facts.

(ii) Assumption of the Previous Question.—The question may be begged indirectly, by assuming that a certain answer has been given to a previous question, from which answer the question now put derives its meaning. This has traditionally been known as the *Fallacy of Many Questions* [*Plures Interrogationes*]. But, as Dr. Mercier points out, "it should rather be called the fallacy of the previous question, for it is the illegitimate assumption of an answer to some question that should have been answered, and has not been, before the problem was stated."[1]

The time-honoured example is 'Have you left off beating your father?' The response you or I would make—'But I never did beat my father!'—shows the nature of the fallacy of demanding a categorical 'Yes or No' as the answer. In disputation such questions seem to have played a not inconsiderable part.

In such a bald form the fallacy seldom meets us now-a-days. Yet, "it is in fact a very frequent form of fallacy, and one that often escapes recognition."[2] It affects the very beginning of a piece of inductive reasoning, for it lies in the

[1] *A New Logic*, p. 387. [2] *Ibid.*, p. 388.

statement of the problem that reasoning sets out to solve. "Why does bread and butter always fall with the buttered side down? is a 'fallacy of many questions' until it is established that it does so fall. What is the connexion between changes of the moon and changes of the weather? is a fallacy of the previous question until it is established that there is a connexion. What is it that makes food that is cooked in copper vessels poisonous? is a fallacy of the previous question until it is established that food so cooked is poisonous. Are the rectilinear markings on Mars canals? is a fallacy of the previous question until it is proved that such markings exist. Why are savage races always cruel? is such a fallacy until it is proved that they always are cruel." [1]

The fallacy may easily be committed in dealing with statistics. In a sense, 'statistics cannot lie.' But unless the preliminary question as to how they are to be used be answered the conclusions drawn from them may be unjustified.

In a book on *The Law of Population*, published in 1830, the author, Mr. M. T. Sadler, collected elaborate statistics of the number of inhabitants to the square mile, and the number of children to a marriage, in different places, and from his arrangement of them aimed at establishing a law that the denser the population the fewer were the births. In reviewing the book Macaulay showed that by arranging the same statistics in a different way an exactly opposite result could be obtained. [2] Mr. Sadler had assumed that his arrangement was the only pertinent one.

To take another example. "In estimating the earnings of labour [in the latter half of the eighteenth century], Young gives many and elaborate tables based upon a preposterous postulate, the unreality of which he admits. The labourer is supposed to be in constant work, and to earn the average of the three seasons, hay-time, harvest-time, and winter, the

[1] *A New Logic*, p. 388.

[2] See two reviews in the *Edinburgh Review* in *Miscellaneous Writings*.

latter being forty-one out of fifty-two weeks. His wife is to work in hay and harvest-time, and to get six weeks' work in winter. His eldest son is to be a first-hand, his second an ordinary hind, his third a farmer's boy, each at the average wages which such a calling supplied. One of his daughters is to be a dairy-maid, and the other a common maid. This family of seven are all capable of work, all at work, and are all collecting their wages into a common fund. He finds the average of such a family's wages to be close to £51 8*s.* a year. Now it is plain that not one family in a thousand corresponded at that time or afterwards to Young's hypothesis, and that, therefore, the calculations based on the statements are, as regards the remuneration of labour, entirely fictitious. What they do represent, and as I believe accurately, is the average cost at which a farmer in the Eastern Counties could procure the services of seven persons in the distribution of occupations in husbandry." [1]

The fallacy is latent in the supposition that the processes of our minds can be investigated and measured by the methods of physics. Before this can be granted, the preliminary assumption that there are mental units corresponding in some definite way with physical units needs to be justified. Not only is this not axiomatic, but many thinkers see objections to it which, in their opinion, render it untenable.

4. Fallacies connected with Process.

(i) Inconclusive Argument [Non Sequitur].

(*a*) "[The writer in the newspaper] is in haste to show that [the body] *was not* kept on shore; for, if so, 'some trace would be found on shore of the murderers.' I presume you smile at the *sequitur*. You cannot be made to see how the mere *duration* of the corpse on the shore could operate to *multiply traces* of the assassins. Nor can I." [2]

[1] Thorold Rogers: *Six Centuries of Work and Wages*, ch. 17.

[2] E. A. Poe: *The Mystery of Marie Roget.*

(*b*) "The average price of corn, it has been said, has fallen considerably since the establishment of the bounty [on its exportation]. . . . . But this event . . . must have happened in spite of the bounty, and cannot possibly have happened in consequence of it. It has happened in France, as well as in England, though in France there was not only no bounty, but, till 1764, the exportation of corn was subjected to a general prohibition."[1]

These examples show the nature of the fallacy. Mere association in the mind, often inspired by some prepossession, is mistaken for cogent reasoning. So what is irrelevant to the conclusion is accepted as proof of its truth. The conclusion drawn simply does not follow from the premises. The whole 'argument' is inconclusive.

To infer an increase of drunkenness from an increase in the number of convictions for that offence before the magistrates in any district may easily be a *non sequitur*. The latter may be due to greater activity on the part of the police, inspired by a Watch Committee of teetotal leanings, to greater severity of the magistrates, or to both combined. Concurrently there may, or may not, be an actual increase of intemperance: but there may, equally well, be a positive decrease. The evidence offered does not establish the conclusion.

The example from Adam Smith illustrates one of the most common forms of advancing as sufficient reason statements which are inadequate. Events that are connected in time—whether as occurring together or in close succession—are imagined, on that evidence alone, to be causally connected [*Cum* (*or post*) *hoc, ergo propter hoc*].

The attribution of a prophetic value to dreams, or to black cats, commits this fallacy as well as that of begging the question. Many people attribute the frequent and heavy rains of 1915 and 1916 to the firing of cannon in Flanders and

[1] Adam Smith: *The Wealth of Nations* (1776), bk. iv., ch. 5.

France. But the firing is much more continuous than the rain, and frequent heavy rain is not unknown in times of peace. The association is not constant; and, if it were, it would still be necessary to show that the firing so affects the atmosphere as to induce rain.

Absence of proof of necessary connexion vitiates all arguments from mere invariable simultaneity or succession, even though further research may establish a causal connexion. Fallacy does not mean that a statement is certainly false, but only that it is not proved to be true by the evidence offered for it. Vendors of patent medicines publish many testimonials from people who have recovered their health after taking them. Until it is established that the medicine in question has a curative effect on the pathological conditions the inference is unsafe. Moreover, the vendors do not obtrude upon public notice cases in which a cure did not follow. As only the favourable cases are advertised the question is begged as well.

(ii) ERRORS IN SELECTION OF EVIDENCE.

(*a*) *Fallacy of Accident* [*Accidens*].—A beginner in the study of demonstrative geometry, in investigating the general properties of triangles draws a figure which, it may be, is approximately an equilateral triangle. In the course of his reasoning he deduces conclusions from the accidental equality of the sides, and imagines them to be true of all triangles. He has introduced into his reasoning relations foreign to the problem upon which he is engaged.

On quite similar lines Rousseau assumed that evils actually existing in French society in the eighteenth century were due to the very existence of society, and drew the conclusion that human nature could be pure only in a supposed 'state of nature,' in which each individual would be independent of the influence of the community.

The argument that social ills of the present day are inevitable, so long as individual property exists, and that,

therefore, they can be removed only by replacing this by collective ownership, is of like nature.

In each case the whole social system is condemned because evils are found in the community, without examination as to whether those evils are due to the system itself, to abuses in its working, or to defects in human nature which a change of system would not remedy. To assume that a desired modification would be the necessary outcome of an advocated change of system is to beg the question.

Or take the following: "Will this mile of railway cost more to lay than that? No, because it is no longer. But is length the only or the chief respect in which resemblance is material to the argument? No. That mile runs on the flat; this must run through a tunnel. That mile runs through agricultural land; this must run through a town. That mile runs over uninterrupted land; this must cross a bridge of wide span. To extract on assured conclusion from such a datum is fallacious."[1]

This mode of taking for proof what is not proof is traditionally known as the *Fallacy of the Accident*, because it consists in taking accidental circumstances, or those that do not determine the conclusion, as pertinent and sufficient.

(*b*) *Confusion of General and Limited Statements.*—A cognate mistake is to regard a general statement, and one limited in time or place or manner, as equivalent to each other. Some solemn kill-joys have acted on it in assuming that because a life given wholly to pleasure is blameworthy, therefore every pleasure is wrong; or, conversely, that because some pleasures are manifestly evil, therefore every pleasure must be condemned.

These are traditionally known as fallacies *A dicto simpliciter ad dictum secundum quid*, and *A dicto secundum quid ad dictum simpliciter*.

[1] Dr. C. Mercier: *A New Logic*, p. 383.

Whenever a general rule is wrongly applied to a particular case, or, conversely, when a particular case is wrongly assumed to fall under a general rule, the fallacy is committed. As a general rule, it is wrong to stick knives into people, but this should not hinder a surgeon from performing an operation, as in the following anecdote of a Russian ambassador at the beginning of the eighteenth century: "His Excellency happened to fall down in a kind of apoplectic fit, when he was paying a morning visit in the house of an acquaintance. The confusion was of course very great, and messengers were despatched in every direction to find a surgeon: who, upon his arrival, declared that his Excellency must be immediately blooded, and prepared himself forthwith to perform the operation: the barbarous servants of the embassy, who were there in great numbers, no sooner saw the surgeon prepared to wound the arm of their master with a sharp, shining instrument, than they drew their swords, put themselves in an attitude of defence, and swore in pure Sclavonic, 'that they would murder any man who attempted to do him the slightest injury: he had been a very good master to them, and they would not desert him in his misfortunes, or suffer his blood to be shed while he was off his guard, and incapable of defending himself.' By good fortune, the secretary arrived about this period of the dispute, and his Excellency, relieved from superfluous blood and perilous affection, was, after much difficulty, restored to life."[1]

The persistence in our own day of the assumption frequently made in the past, not only by irresponsible writers but also by responsible statesmen, that the introduction of English institutions will cure all the ills of the body politic in every country on earth, regardless of its history, traditions, and social structure, is a striking proof that this fallacy is by no means obsolete.

[1] Sydney Smith: *Peter Plymley's Letters*, Let. 9.

5. Neglect of Evidence.

(i) In Application of Theories to Practice.—All thinking deals with matter selected from the great world of fact. It is abstract, because it omits from consideration all that it believes to be irrelevant to the purpose in hand. Hence arises the danger that it should apply its abstract conclusions to concrete facts without allowing for the influence of the factors it has disregarded in its inference.

The laws of motion are valid conclusions as to the action of physical forces considered abstractly. The universal fact of friction is disregarded. But in applying those laws in the making of a machine the influence of this omitted element must be allowed for, or the machine will not work.

Similarly, theories of ethics which take account only of reason, or of the desire for pleasure, as guides of conduct, have little value for real life, in which so many factors cooperate in deciding us how to act.

The political economy current in the first half of the nineteenth century was an abstract system, built up on the conception of the 'economic man'; that is, one whose actions were solely determined by desire for wealth, who was in equal and free competition with all other men, and able to turn with equal facility to any occupation. All modifications of economic activity due to other springs of action except laziness or desire for present enjoyment,—such as charity for others, force of custom, vengeance, or stupidity—were ignored. Had the results been regarded as equally abstract with pure physics, they would have had an intellectual interest. Unhappily they were applied in much mistaken legislation, without any such modifications as are made in the theories of physics when they are applied in machinery.

Always it should be borne in mind that "arguments about the abstract cannot handle and determine the concrete."[1]

[1] Card. Newman: *The Grammar of Assent*, p 271.

Often we hear 'Oh! that's all very well in theory, but it won't do in practice.' The theory the objector has in mind is just such abstract theories as these. A true theory of practice modifies abstract conclusions by consideration of the circumstances in which the practice has to be done. It takes into consideration those very elements which the abstract theory omits. It recognises that everything which may affect practice is pertinent to a practical question.

(ii) IN ESTIMATION OF CIRCUMSTANTIAL EVIDENCE.—Even if all the evidence be received, its total weight may be wrongly estimated. Indirect evidence is known as 'circumstantial.' Its force consists in the concurrence of independent pieces of evidence all pointing to one conclusion, which by their variety and consistency give a strength of probability which for practical purposes we accept as proof. Indeed, it is often the only proof available. A burglar or murderer avoids that observation of his deeds by his fellows which would make direct evidence against him available. Much of historical investigation must trust mainly to circumstantial evidence.

In dealing with such evidence two fallacies are possible.

First. Each piece of evidence may be considered by itself, when its persuasive force is small, and the added strength due to their combination be ignored. They are added together, whereas the mutual support they give to each other makes their combination a product of factors, rather than a sum of units. Of this cumulative force the evidence adduced in favour of the belief that the *Letters of Junius* were written by Sir Philip Francis is a good illustration.[1]

Second. The cumulative evidence may be regarded as furnishing a certain proof. But we have proof only when no alternative is possible, and circumstantial evidence cannot ensure this. The elements may be merely coincidences, so

[1] See pp. 153-155.

that their apparent cumulative force is illusory. At the most such evidence can make any alternative highly improbable. So it is laid down that "In criminal prosecutions, the circumstantial evidence should be such, as to produce nearly the same degree of certainty as that which arises from direct testimony, and to exclude a rational probability of innocence."[1] This is what is meant by 'practical proof': no avoidance of the conclusion seems possible.

On such practical proof—that is, on a high degree of probability—we have constantly to act. Many an unwise decision and unjust sentence has borne witness that such action may be based on error.

These are all logical questions. That is a defective logic which supposes that all thought can be reduced to rule, and refuses to consider any arguments save those that lead to a formally demonstrated conclusion. It must be acknowledged that such unjustifiable limitation of its scope was long traditional. But the defective use of logic is certainly not a reason for the neglect of logic; it is a call for a more adequate use of it. With logic, as with other good things, "the abuse taketh not away the use thereof."

[1] Phillipps: *Law of Evidence*, vol. i., p. 456 (quoted by Newman, *op. cit.*, p. 316).

CHAPTER VIII.

THE STUDY OF FACTS.

1. Observation demands Skill.

It is related that in a certain continental legislature a proposal to vote a grant for the provision of technical instruction in agriculture was strongly opposed by the land-owners on the ground that it was unnecessary. The proposer quietly asked these vociferous objectors if any of them would be good enough to tell him whether a cow's horns are behind, or in front of, her ears. Not one took up the challenge, and the vote was agreed to without further discussion. But it was noticed that the propounder of the query which had overthrown his adversaries did not himself supply the answer.

Can the reader do so? If not he must be convinced that he has frequently seen what he has never noticed. If he *can* answer this particular question, he may easily find others which will bring home to him the same truth. In what order does a cat move its legs in walking? What is the exact pattern of the carpet in your dining-room? How many windows are there in the Town Hall?

We might go on indefinitely. It soon becomes evident, even to the most incredulous, that our knowledge of many familiar objects is sufficiently *clear* to enable us to recognise them, but not *distinct* as to many of their details.

Seeing, then, is not observing. That is why we make so many mistakes as to what we have seen. No adage deserves less respect than 'Seeing is believing.' In many cases we should be nearer the truth were we to reverse it, and say

'Believing is seeing.' For we often 'see' what we believe to be before our eyes when in truth something quite different is there.[1]

It follows that observation is a work of skill, and has to be learnt.

2. **Observation is Selective.**

Out of the immense mass of impressions on our senses we pick out some to study with attention. That study is observation.

What we so select is determined by our interests. A tourist who is interested in architecture will study buildings, choosing those that are of most worth as characteristic of various periods, or as serving some definite purposes. It may be that in this general interest he has prominent in mind at the moment a more particular interest, and so confines himself to churches, or, yet more narrowly, to churches of a definite period, say, the early Gothic. If he be himself an architect and have in hand the designing of a new church, his observation will be determined in detail by this purpose. He will attend most to what is cognate to it and so helpful to him in his work, and will pass by with much less intense observation features that do not fall in with his scheme.

Another traveller over the same ground may be a botanist. To him the objects of interest will be quite different. He will study plants and trees and flowers, and if he be at the time working at some species of plant, examples of that will most engage his attention.

Another may be a student of sociology; to him the arrangements of political and social life will be of primary interest.

Yet another may be a gourmet. He will give close observation to the meals served at the hotels and restaurants he frequents. And so on.

[1] Cf. p. 99.

Doubtless, any one person may be interested in more than one class of experiences. But the rule holds that what each observes is determined by his interests.

3. **Specialisation of Interests.**

To things that do not interest us we pay no more attention than is demanded by the practical use we may wish to make of them. Interest is always specialised.

We begin by being open to all sorts of impressions. To all we give as much notice as is necessary to make use of them. Out of this general indeterminateness, determinate interests grow, according to our tastes working in our surroundings. When an innate taste is very strong it seeks for food. That we see in the struggles of genius amid uncongenial surroundings, or forced by outside influences into occupations which yield it no scope. But most men have no such overpowering impulse towards a definite line of activity. They make the best of the circumstances in which they find themselves, and, often only half consciously, select among them those that win their liking, and so call forth their best and most fruitful efforts.

Each one, then, tends to become specialised in both thought and action. As he thinks and acts in his chosen lines, his knowledge in those lines increases both in range and in depth. But such increase is gained at the cost of relinquishing both hope and desire for knowledge outside that limit.

So we all become in a sense specialists in some few departments of human activity. Knowledge and skill even in these may be small, but they are yet smaller in other departments. On the other hand, they may be large. Then we have the great artist or discoverer or inventor, the great statesman or administrator. The difference is in the powers to observe, that is, to select and study all that is pertinent to our work, and to organise the results of our investigations into valid systems of thought.

4. Observation and Thinking.

Often skill in the study of facts, and skill in using them in inference, are not combined in equal strength in the same individual. In the progress of knowledge some men have done their best work in collecting facts; others, it may be long afterwards, in seeing into the meaning of those facts. "Take the case of a man like Stephen Groombridge, who made thousands of accurate observations of stars in the early part of last century. Fifty years later something of the value of his work began to emerge from a comparison with later observations which showed what stars had moved and how; but it was not until nearly a century had elapsed that something about the laws of stellar movement was extracted from his patient work, combined with a repetition of similar work at Greenwich."[1]

But, whether done by one mind or by many minds, the two forms of work are equally necessary to that building up of true systems of thought in which the advance of knowledge consists.

Though some men can observe better than they can combine their observations into systems, yet observation itself involves thought.

In the first place, it is selective. Interest determines the general line of our observations; our immediate purpose decides them in detail. Mere interest, however, will not guide us in what we select for study. That depends on what seems to us to be pertinent, and that is an inference from our knowledge of the matter in hand.

We have seen that this selection may be imperfect, by the omission of something which should be taken into account, by the inclusion of something which is not really material, or by the distortion of facts, either unintentionally, or deliberately, to make them fit into some preconceived theory.

[1] Prof. H. H. Turner: "Address at British Association, 1911," *The Times*, Sept. 1, 1911.

This possibility of error shows that observation is not simply recognition of the existence of facts; it is the choosing and interpretation of them. The best observer is he who starts with a question to answer, but with no prejudice in favour of one answer rather than another. The question may be whether a certain theory be true or false, or it may be simply 'What exactly are these facts?' In the latter case bias towards some one answer is less likely to be present; in the former it has to be specially guarded against.

In the second place, the decision as to what a fact *is* depends on inference from past experience. In the simplest case of familiar recognition we decide, for instance, that a small moving object, so high up in the sky that we cannot clearly make out its shape, is a bird, because we have had many experiences of seeing birds flying at various heights, and becoming more indefinite in shape as they soar higher. From similarity of appearance we infer identity of nature. But we may be wrong; it may be an aeroplane.

The less familiar and the more indistinct is the object observed, the easier is it to be mistaken in deciding what it is. The following startling example brings this home to us without further comment.

"THE ROKEBY 'VENUS.'—The committee invited by the Editor of the *Morning Post* to investigate the supposed signatures on the Rokeby 'Venus,' having met twice to examine photographs and lantern slides submitted by Mr. James Greig and Mr. W. E. Barber, and having heard full explanations from those gentlemen of the markings discerned by them, and having also, through the courtesy of Sir Charles Holroyd and the Trustees of the National Gallery, had opportunities of examining the picture itself on two separate days without the glass and in strong daylight, have issued the following report:—

I. THE B. M. SIGNATURE.—Six members of the committee are of opinion that the markings indicated by Mr.

Greig are not deliberate, and do not form a signature. Three members believe the marks to be intentional, but do not commit themselves to the reading of them given by Mr. Greig. One member is not prepared to express an opinion.

II. The Mallet.—Six members are satisfied that there is no deliberate inscription at this point. The remaining members are prepared to admit the possibility of the existence of a deliberate inscription.

III. The Date.—The committee agree that in the place where Mr. Greig and Mr. Barber see a date there are markings which resemble figures. The majority of the committee believe these markings to be deliberate, but they differ as to the reading of them.

IV. The Mengs Signature.—No member of the committee could see any recognisable inscription. Some members see markings, but differ as to whether they are intentional.

V. The Erasure.—Two members of the committee believe that there are signs of erasure in the part of the canvas contiguous to the markings under discussion. One member sees signs of an abrasure, but expresses no opinion as to whether it is deliberate. Four members are definitely of opinion there has been no deliberate erasure and that the appearances are to be explained otherwise. Three members, not having expert experience, express no opinion.

(Signed) Frederic G. Kenyon (Chairman), Sidney Colvin, Hugh Lane, Solomon J. Solomon, J. M'Lure Hamilton, Walter Sickert, A. J. Finberg, William McKay, W. G. Rawlinson, Fabian Ware.

April 29, 1910."

5. Use of Scientific Instruments.

That accurate observation calls into play knowledge acquired from previous experience is evident whenever it is

made with the aid of scientific instruments. Little astronomical work can be done without the telescope; much biological and geological observation requires the microscope; physics and chemistry cannot get on without the balance and the spectroscope.

But if we ask ourselves what a scientific instrument is, we must answer that it is a system—and often a very complex system—of knowledge embodied in such substances as brass and glass. In using the instruments we bring to bear on our observation all the knowledge involved in their structure.

It is not surprising, therefore, that one has to learn to use them. The tyro cannot apply this embodied knowledge, because he has not made it his own in any way. Only when one knows it, at least in its practical bearings, can one use the instruments effectively. So, "skill in modern laboratory work is as far out of the reach of the untaught as performance on a musical instrument."[1]

In all observation we select; in all selection we apply knowledge and use inference; and throughout we are guided by our purpose.

With continued observation grows the power of discrimination. The expert can distinguish shades of difference which are imperceptible to the ordinary man. An artist sees many shades of colour between two tints which to the ordinary eye are identical; a musician can detect differences of pitch and tone which are imperceptible to the untrained ear; a tea-taster or cigar-taster can range in an order of merit samples which, to him who is not a connoisseur, are of equal quality.

So with the skilled thinker. To him distinctions in thought are "gross as a mountain, open, palpable" which the man in the street does not recognise at all, or dismisses as making no difference.

[1] Sir T. Clifford Allbutt: Article on "Medicine" in *Ency. Brit.*, 11th edn.

By the aid of instruments the power to detect small differences is enormously increased. Step by step with it goes increase in exact knowledge; without it knowledge remains at the elementary stage of ordinary life, in which all distinctions except those that lie on the surface are ignored.

6. Experiment.

(i) Nature of Experiment.—Now we can take another step forwards. Having grasped the truth that many differences which are hidden from the untrained mind are of the greatest importance, we reach the problem of the possibility of isolating them, and of submitting their occurrence to our control. True, we cannot make the facts that happen in the world other than they are, but cannot we select the facts we study, not only in thought but in their real happening?

The answer to this is experiment, which is simply observation of an event the occurrence of which is determined by the observer.

It seems to be a common belief that experiment is something done in a scientific laboratory with the aid of elaborate instruments. Certainly some experiment is of this character. But it need not be so. In ordinary life we are always making experiments. The boy who plays a trick on his father or his schoolmaster 'to see how he will take it' works an experiment, and by its results gets knowledge that he may find useful for future guidance. Again he experiments if, when set to work a sum he does not clearly understand, he tries the application of some rule in the hope that it will produce the correct answer.

If I wish to decide whether I can learn a stanza of poetry more easily by reading it straight through again and again till I can repeat it, or by memorising each line separately, I try the experiment with two stanzas of equal length and similar construction, and I note the time the acquisition takes in each case.

If a person suffering from some disease is led to dose himself with a patent medicine, he works an experiment on his own state of health. Often, it may be presumed, his physician also experiments on that state, but he does it with much greater knowledge of the nature both of the trouble and of that of the drugs he administers. So his experimenting is likely to be more to the purpose. He may, for example, be sure that his patient needs a tonic, but may have to try several times before he finds out which one is the most beneficial.

Experiment, then, is no more a matter of the use of instruments than is simple observation.

Still less is it a matter of the kind of inquiry on foot.

It is observation with a purpose so definite, and with pertinent knowledge so adequate, that the occurrence of the event, and the conditions under which it occurs, can be determined by the observer.

(ii) Aim of Experiment.—Experiment is evidently not possible in every investigation. The astronomer cannot control the motions of the stars, the geologist cannot make the past history of the earth other than it has been.

But the chemist can experiment, and in no branch of knowledge has experiment played a larger part than in chemistry. Chemical action can be studied on a small scale and under very definite conditions. Its results are quickly reached and are definite. This is because chemistry deals with only one class of natural forces, so that, though it works with material things, the relations it establishes are abstract.

In some other fields of investigation the isolation of the agents to be studied and of the results they bring about cannot be secured. So the nature of the results does not stand out clearly. For example, every change in the laws is an experiment on the well-being of the community. But the action of any particular law cannot be separated from other

factors in social life, many of which are sure to modify the effect it would have if they were not present, or were other than they are.

The simpler the matter in hand and the more nearly the analysis of fact by thought can be reproduced in the facts themselves, the surer and more potent a means is experiment for the solution of definite problems and the answering of definite questions.

(iii) Analysis of Experiment.—The essence of experiment is the planning what shall be done in order to secure that a definite event happens under well-known conditions, so that the change wrought can be studied by itself.

The use of instruments adds precision to experiment, and in many cases is indispensable. But instruments are only adjuncts. It is not the manipulation of instruments which makes the experiment, but the planning of what is to be done either with or without instruments. The great experimenter in science often does not work out his experiments in the laboratory. He invents them, and leaves the mechanical execution to people whose time is less valuable than his own.

The actual carrying out of the planned experiment is analogous to the work of the builder; that of the deviser of the experiment to that of the architect. In each case the latter is the directing mind; the former is merely the operating hand.

(iv) Value of Experiment.—Experiment can ensure the occurrence of an event at any time. So the observation can be repeated under exactly the same conditions as often as is desirable. Thus, the observation of one investigator can be tested and checked by another.

It can do more than this. It can secure the happening of events under conditions which nature does not supply. "Experiment *invents* original phenomena which nature left to herself never realises; for example, the fall of bodies in a vacuum, the liquefaction of hydrogen and oxygen. The

chemist creates in his laboratory many compound bodies which do not exist outside it."[1]

(v) DIFFICULTY OF EXPERIMENT.—Whether observation be simple, or under the semi-artificial conditions of experiment, the observer has to be on his guard that he does not assume that whatever he has not noticed does not exist. For long years experiments had been made on the composition of the atmosphere before such gases as argon were found in it. Until every factor in the event is accounted for, the work of the investigator is unfinished. And many factors are so small that they may well escape recognition for a long time.

7. Summary.

It is evident, then, that scientific, or exact, observation is work which demands the greatest skill, that it can only be done well by those who have devoted much time and energy to acquiring the power, and that then its effectiveness depends not only on the care, but also on the knowledge, of the investigator.

The devising of an experiment to decide the answer to a certain question demands in addition much constructive power. The experimenter must see in imagination the conditions it is necessary to secure, and then bring to bear all his ingenuity to plan means that will secure them.

Frequently he has to invent his instruments before he can use them. This, too, is inference based on knowledge, for every new instrument is developed out of one already existing, but not altogether suitable for the work in hand. Thus we see that the advance of knowledge calls for the invention not only of systems of thought, but of all the material aids by which the relations which bind together those systems are brought under observation.

[1] Rabier: *La Logique*, p. 115.

CHAPTER IX.

INDUCTIVE DEALING WITH FACTS.

1. Facts must be reduced to System.

We study facts that we may learn their relations to each other and to ourselves. A fact by itself means nothing to us. It is only when we see it as a constituent element of some system that we understand it.

The relations we know and express are as innumerable as the things they bind together. But, like them, they may be grouped into wide general classes. Of these the chief are—

(1) *Likeness and Unlikeness.*—The first step towards knowledge is the grouping of like with like, and the separation of the unlike. On it rests all classification. The relation is between qualities.

(2) *Class Inclusion and Exclusion.*—To group like with like is to see each as a member of a class. Moreover, classes are grouped in a similar way, and so are thought as members of a wider class. This gives a relation between things—one of denotation.

(3) *Subject and Attribute.*—All grouping rests on the recognition that objects have qualities. So is brought out the relation of subject and attribute. This is a relation of connotation to denotation. On it is based definition.

(4) *Affinity.*—We recognise between members of the same family—whether of human beings, of animals, or of plants—relations based on a common descent. A consideration of this may determine the points of likeness taken as the ground of classification, and expressed in definition.

(5) *Quantity.*—We compare things and groups of things on the basis of more and less. When we do this by measurement—that is, by counting the number of a chosen unit in each—we have definite *Numerical Relations.* When we do not pass beyond the recognition of more and less in size or in some quality, we establish *Relations of Degree.* In every case we make a serial arrangement on the basis of quantity.

(6) *Space.*—We know all things as existing, and all events as occurring, in space. We may consider their spatial relations apart from the things and events themselves, and thus construct a system of geometrical relations, such as is expressed in Euclid's 'Elements of Geometry.'

(7) *Time.*—Relations of time give form to records of events. They all fall under the two heads of *Co-existence* and *Succession.*

(8) *Causality.*—We cannot think the course of the world as accidental. Hence, the most important relation for knowledge of events is that of causation, which shows that the occurrence of one event leads inevitably to another event. The former we call the *Cause* of the latter; the latter the *Effect* of the former.

As all events occur both in time and in space, those relations are always present with relations of causation. To assume that when they are established causation is also established is to fall into the fallacy of *post* [or *cum*] *hoc, ergo propter hoc.*[1]

2. **Generalisation.**

The facts studied are few ; those in the universe are innumerable. Yet on the ground of observation of those selected facts general relations are established. The particular facts are regarded as typical, so that results obtained from them can be generalised.

[1] See pp. 128-129.

The generalisation is to the limits of the class to which we assign them. It follows that the value of all empirical laws depends on the soundness of classification. A good classification is, then, the first step in an inductive inquiry into the constitution of the world. But this involves accurate knowledge of the nature of things, as its ground.

Exact knowledge, however, is developed out of less perfect knowledge, and the process of development is often a very slow one. However vague and imperfect the knowledge it implies, some arrangement of facts into classes is presupposed in all thought. Without it the world would be a chaos, and experience could not begin to teach us. We assume from the first that things called by the same name are of the same nature. We trust to the classification which the long experience of our ancestors has embodied in the language we learn; and, for the practical purposes of daily life, we find this guidance sufficiently trustworthy.

It is under the goad of some special need or interest that we set to work to examine critically the current classification, to make it more accurate, and to extend the same principle of arrangement to facts which do not come into the experience of every-day life and to elements into which we are led to analyse common things.

3. **Science.**

Without waiting to obtain exact—or scientific—knowledge of the facts of common life we are continually urged by daily needs to weave them into systems. Induction in science is not a new mode of reasoning. It is only the kind of reasoning we constantly use in daily life, in matters of very small importance as well as in those of greater moment, made as accurate as possible. We cannot fix a boundary line between 'scientific' and ordinary thought and knowledge. There is a continuous scale, ranging from the vague beginnings of knowledge in childhood to the very definite knowledge in

some range of experience possessed by the man who has devoted his life to research. The former is at once seen to be 'unscientific' and the latter is undoubtedly 'scientific.' But in the middle region it would often be a bold thing to apply either adjective.

Whatever the matter we study, as thought grows in accuracy it becomes increasingly scientific. For 'science' is exact knowledge of any kind. The limitation of the name to certain fields of knowledge is itself an instance of unscientific popular thought. And 'popular' thought is the thought of people who have given no real study to the subject in hand, whatever it may be.

4. Examples of Induction.

The facts of experience are always challenging us to explain them. Many of these challenges we ignore. But when any of them touch our interests we try to answer the question they raise in our minds. The answer can be found only by referring each to its appropriate place in a system. When this is so done that each points to the same answer we feel sure that we have solved the problem.

(*a*) *A Domestic Incident.*—For example, I find the inkpot on my desk overturned, and the pages of my manuscript stained with ink. The question is at once raised of how the mess was made. Two solutions may present themselves to me as possible. The servant may have overturned the inkpot in dusting the table, or the cat may have been roaming at large in my study, have jumped on the table, and knocked over the inkpot.

If the servant is a careful person, she is so far unlikely to have been the agent; and if she had accidentally done the mischief she would have been likely to have set up the inkpot again, and to have wiped up the spilt ink. Further, if she is accustomed to tell me when she has accidentally done any mischief, and though she has seen me this morning has said

nothing about it, the probability that she was concerned in the trouble is decreased.

So the supposition that the cat was the guilty individual is indirectly increased. If then it is found that the cat has undoubtedly visited the study, and if she is known to be in the habit of jumping on tables, the case against her begins to look black; if on examination ink is found on her coat she stands condemned. The problem is solved, the facts all support one conclusion, and I am as fully convinced that the cat overturned the inkpot as if I had seen her do it.

(*b*) *Unravelling a Crime.*—Such simple pieces of inductive reasoning are of daily occurrence. They differ only in simplicity from the inductive inquiries by which a crime is brought home to its perpetrator. The student of logic may get much insight into the methods of inductive reasoning by studying the tales of the detection of criminals written by Edgar Allan Poe, or those studies in mystery which Sir Arthur Conan Doyle has given us in the exploits of 'Sherlock Holmes.' In all, the inferences are inductive, or 'analytic,' as Poe rightly calls them. Conan Doyle obscures their real nature by using 'deduction' as synonymous with inference.

Let us, as an illustration, briefly set out the main points in Poe's *Murders in the Rue Morgue.*

A mother and daughter lived together in the fourth story of a house, otherwise unoccupied.

About three o'clock one morning the inhabitants of the district were aroused from sleep by a succession of terrific shrieks, issuing apparently from that story. "After some delay, the gateway was broken in with a crowbar, and eight or ten of the neighbours entered, accompanied by two gendarmes. By this time the cries had ceased; but, as the party rushed up the first flight of stairs, two or more rough voices, in angry contention, were distinguished, and seemed to proceed from the upper part of the house. As the second landing was reached, these sounds, also, had ceased."

On reaching the fourth floor the doors of the two rooms of which it consisted were found locked, and the keys in the locks inside the rooms. Forcing open one of the doors, the searchers saw a room in great disorder, "the furniture broken and thrown about in all directions. . . . On a chair lay a razor, besmeared with blood. On the hearth were two or three long and thick tresses of grey human hair, also dabbled in blood, and seeming to have been pulled out by the roots." A considerable sum of money lay on the floor.

Further search discovered the corpse of the daughter pushed up the chimney, head downwards, and on the throat were "dark bruises and deep indentations of finger nails."

Nothing else was found in the house, which was furnished only on the fourth floor. But in a small paved yard at the back, "the corpse of the old lady, with her throat so entirely cut that, upon an attempt to raise her, the head fell off," was discovered. The whole body was badly bruised, and many of the bones were shattered.

At the inquest, all the people who had entered the house agreed that one of the voices heard as they were going up the stairs spoke in French. The other voice was described as "shrill" or "harsh," but none had understood what it said. The witnesses were of different nationalities, and each was "sure that it was not the voice of one of his own countrymen." One Frenchman supposed it to be that of a Spaniard, another that of an Italian; a Dutchman that of a Frenchman; an Englishman that of a German; an Italian that of a Russian. There was only one point in common: each attributed it to a language of which he confessed he had no knowledge.

Here, then, was a problem. Two murders had been committed. Who was guilty?

The police were completely at fault. Robbery could not be the motive, for the money was left.

The problem was solved by the Chevalier Dupin, by examination of the facts and keen analytic reasoning upon them.

First, he selected those facts that were pertinent. He scrutinised the outside of the house, both back and front; he examined with care both the bodies of the victims and the interior of the two rooms.

The first question he set himself to answer was: How did the murderer or murderers get out without being seen? They could not have passed through either of the locked doors. Minute search showed there was no secret entrance. The chimneys at the height of some eight or ten feet narrowed so that a large cat could not get through. Nothing was left but the windows. "Through those of the front room no one could have escaped without notice from the crowd in the street. The murderers *must* have passed, then, through those of the back room."

Of these there were two, but in the frame of each a large nail had been inserted nearly up to the head. The police failed to raise them, and "were entirely satisfied that egress had not been in these directions." But Dupin's reasoning proved that it must have been. He, therefore, examined the windows minutely, and found that, in addition to the nail, each was held by a spring. One nail was intact, but the other was broken, so that when the spring was released that window could be raised. This, then, *must* have been the passage. Doubtless, too, through this the murderers had entered.

How get from it to the ground? About five feet and a half from the casement in question was a lightning-rod, and to the window was fitted an iron shutter some three and a half feet broad, the lower half of which was in open trellis-work. To effect this passage would demand a very unusual degree of activity. Here was the first characteristic of the murderer. The injuries inflicted indicated, besides, an equally unusual strength and ferocity. Here, then, was a second characteristic.

Now comes in the bearing of the discrepant evidence as to the language in which the harsh voice had spoken. It could

not be a European language, and the presence of an Asiatic or African was highly improbable.

The one explanation that would harmonise all the facts with each other and with other pertinent knowledge was that the murders had been committed by an escaped ourang-outang, and that the French voice was that of its owner, who was trying to re-capture it. He was probably a sailor, or he would have been unlikely to have been able to climb up the rod. Horrified at what he saw, and terrified lest he should be accused, he would have left the beast, returned home, and withheld his evidence. The ape would then have escaped.

By a cunningly worded advertisement of the finding an ourang-outang at large some miles away, Dupin induced the sailor to call on him, and elicited a confession that confirmed his theory in every point.

(*c*) *Who was* 'JUNIUS'?—As an example of inductive reasoning on a quite different matter we will take the attempts to solve the problem of the authorship of the *Letters of Junius*.

"The wildest guesses as to his identity were made in his own day and after. It was thought at first that only Burke could write so well, and most of the eminent contemporaries of JUNIUS have, at one time or another, been charged with the authorship of the letters."[1]

Macaulay thus sums up the evidence in favour of the supposition that the letters were written by Sir Philip Francis. "Was he the author of the Letters of JUNIUS? Our own firm belief is that he was. The evidence is, we think, such as would support a verdict in a civil, nay, in a criminal proceeding. The handwriting of JUNIUS is the very peculiar handwriting of Francis, slightly disguised. As to the position, pursuits, and connexions of JUNIUS, the following are the most important facts which can be considered as clearly

[1] C. W. Previté-Orton in *Cambridge History of English Literature*, vol. x., p. 407.

proved: first, that he was acquainted with the technical forms of the secretary of state's office; secondly, that he was intimately acquainted with the business of the war-office; thirdly, that he, during the year 1770, attended debates in the House of Lords, and took notes of speeches, particularly of the speeches of Lord Chatham; fourthly, that he bitterly resented the appointment of Mr. Chamier to the place of deputy secretary-at-war; fifthly, that he was bound by some strong tie to the first Lord Holland.

Now, Francis passed some years in the secretary of state's office. He was subsequently chief clerk of the war-office. He repeatedly mentioned that he had himself, in 1770, heard speeches of Lord Chatham; and some of these speeches were actually printed from his notes. He resigned his clerkship at the war-office from resentment at the appointment of Mr. Chamier. It was by Lord Holland that he was first introduced into the public service.

Now, here are five marks, all of which ought to be found in JUNIUS. They are all five found in Francis. We do not believe that more than two of them can be found in any other person whatever. If this argument does not settle the question, there is an end of all reasoning on circumstantial evidence.

The internal evidence seems to us to point the same way. The style of Francis bears a strong resemblance to that of JUNIUS; nor are we disposed to admit, what is generally taken for granted, that the acknowledged compositions of Francis are very decidedly inferior to the anonymous letters. The argument from inferiority, at all events, is one which may be urged with at least equal force against every claimant that has ever been mentioned, with the single exception of Burke; and it would be a waste of time to prove that Burke was not JUNIUS. And what conclusion, after all, can be drawn from mere inferiority? Every writer must produce his best work; and the interval between his best work and

his second best work may be very wide indeed. . . . Nay, it is certain that the Man in the Mask, whoever he may have been, was a most unequal writer. . . .

One of the strongest reasons for believing that Francis was JUNIUS is the moral resemblance between the two men. It is not difficult, from the letters which, under various signatures, are known to have been written by JUNIUS, and from his dealings with Woodfall and others, to form a tolerably correct notion of his character. He was clearly a man not destitute of real patriotism and magnanimity, a man whose vices were not of a sordid kind. But he must also have been a man in the highest degree arrogant and insolent, a man prone to malevolence, and prone to the error of mistaking his malevolence for public virtue. . . . It may be added that JUNIUS, though allied with the democratic party by common enmities, was the very opposite of a democratic politician. . . . All this, we believe, might stand, with scarcely any change, for a character of Philip Francis."[1]

More recent critics are not as confident as Macaulay. Mr. Previté-Orton thus sums up the present position: "The letters of JUNIUS seem to be brought home to a small group which included Calcraft, Francis and, perhaps, Lord Temple. They passed through Francis's hands, and he is their most likely author. He evidently wished to be thought so; but, if he was, the malignant talent they displayed could only develop in secrecy, or, perhaps, his prime was short. He remains in his real character a pretender only, in his assumed, a shade: *stat nominis umbra.*"[2]

This example well brings out the cumulative force of circumstantial evidence, each piece of which is, by itself, of little weight. It also shows that inductive inference cannot always yield a certain proof, but only a probable conclusion.

(*d*) *Hiero's Crown.*—We will now take an example from the early history of science.

[1] *Essay on Warren Hastings.* [2] *Op. cit.*, p. 410.

"Hiero, king of Syracuse, had given a lump of gold to be made into a crown, and when it came back he suspected that the workmen had kept back some of the gold, and had made up the weight by adding more than the right quantity of silver; but he had no means of proving this, because they had made it weigh as much as the gold which had been sent. Archimedes, puzzling over this problem, went to his bath. As he stepped in he saw the water, which his body displaced, rise to a higher level in the bath, and to the astonishment of his servants he sprang out of the water and ran home through the streets of Syracuse almost naked, crying *Eureka! Eureka!* ('I have found it, I have found it.')

What had he found? He had discovered that any solid body put into a vessel of water displaces a quantity of water equal to its own bulk, and therefore that equal weights of two substances, one light and bulky, and the other heavy and small, will displace different quantities of water. This discovery enabled him to solve his problem. He procured one lump of gold and another of silver, each weighing exactly the same as the crown. Of course the lumps were not the same size, because silver is lighter than gold, and so it takes more of it to make up the same weight. He first put the gold into a basin of water, and marked on the side of the vessel the height to which the water rose. Next, taking out the gold, he put in the silver, which, though it weighed the same, yet, being larger, made the water rise higher; and this height he also marked. Lastly, he took out the lump of silver and put in the crown. Now if the crown had been pure gold, the water would have risen only up to the mark of the gold, but it rose higher and stood between the gold and silver mark, showing that silver had been mixed with it, making it more bulky; and, by calculating how much water was displaced, Archimedes could estimate roughly how much silver had been added."[1]

[1] Miss A. B. Buckley: *A Short History of Natural Science*, pp. 22-23.

This problem is a simpler one than either of the last two, for it involved only a relation of inanimate matter. Moreover, the analogy suggested to Archimedes between the effect of immersing his own body and another body could be tested by a simple experiment. The solution of the particular problem was a certain one. But, more important than that, a general law of nature was discovered from which many conclusions could be deduced. The example also shows the value of measurement in giving greater precision to experiment.

(*e*) *A Pre-historic Painting.*—As a last example we will take an instance from anthropology.

In the Little Pyrenees is the cave of Niaux, "high up in a valley scarred nearly up to the top by former glaciers. This cave is about a mile deep; and it will take you half a mile of awkward groping amongst boulders and stalactites, not to mention a choke in one part of the passage such as must puzzle a fat man, before the cavern becomes spacious, and you find yourself in the vast underground cathedral that pre-historic man has chosen for his picture-gallery. . . .

There are . . . whole screeds of symbols waiting, perhaps waiting for ever, to be interpreted. The dots and lines and pothooks clearly belong to a system of picture-writing. Can we make out their meaning at all? Once in a way, perhaps. Note these marks looking like two different kinds of throwing-club; at any rate, there are Australian weapons not unlike them. To the left of them are a lot of dots in what look like patterns, amongst which we get twice over the scheme of one dot in the centre of a circle of others. Then, farther still to the left, comes the painted figure of a bison; or, to be more accurate, the front half is painted, the back being a piece of protruding rock that gives the effect of low relief. The bison is rearing back on its haunches, and there is a patch of red paint, like an open wound, just over the region of its heart. Let us try to read the riddle. It may well embody a charm that ran somewhat thus: 'With these weapons, and by these

encircling tactics, may we slay a fat bison, O ye powers of the dark!' Depend upon it, the men who went half a mile into the bowels of a mountain, to paint things up on the walls, did not do so merely for fun. . . . In Australia, . . . rock-paintings of game-animals, not so lifelike as these of the old days, but symbolic almost beyond all recognising, form part of solemn ceremonies whereby good hunting is held to be secured. Something of the sort, then, we may suppose, took place ages ago in the cave of Niaux."[1]

5. The Inductive Method.

Let us now ask ourselves what are the characteristics of method common to all these examples of inductive inference.

(i) Collection of Facts.—The first stands out clearly. They all start from careful observation of facts, and selection of those that seem to bear on the problem. Facts which appear to be irrelevant are put on one side. In Poe's tale Dupin eliminated the doors as possible means of egress, because each was found locked, with its key inside the room, when the searchers reached it. He put on one side the chimneys because they were too narrow; the front windows because they were under observation.

The decision as to what facts are really pertinent is in many cases one of extreme difficulty, but a mistake in it vitiates the whole inference. The clearer the grasp of what the question at issue is, the more likely is the selection to be a good one. In addition to this there are personal qualities: the open mind, as contrasted with the mind already made up which allows its choice to be guided by its preconceptions; the mind well stored with cognate knowledge, with many precedents and like situations at command. Behind these, and directing them, are insight and analytic power, the products partly of natural endowment and partly of cultivation. In these people differ widely.

[1] R. R. Marett: *Anthropology*, pp. 50-52.

(ii) Supposition, or Colligation of Facts.—When the facts are selected a relation must be imagined which binds—or colligates—them into a system. We then arrange them in thought according to this supposed connexion, and see whether they hang together consistently.

Two or more such suppositions may present themselves on a first view. If all the known facts are consistent with both, fresh facts must be sought, till we find some that will fit into only one of the imagined systems. These are *crucial instances*, enabling us to reject all but one of the suppositions.

The process is illustrated in our examples. There were many guesses as to the identity of Junius. Only a minute examination of contents and style, and comparison with the sources of information, character, and style, of any suggested author could bring these suppositions to the test of fact, and from that process the supposition that Junius was Francis emerges as the most probably true.

The possible method of testing the supposition that silver had been partially substituted for gold in the royal crown was suggested to Archimedes by observation of a fact. There was no novelty in this fact, to him or to others. What was new was the seeing its bearing on the problem. Masses of metal and the human body were recognised as being alike in this one relation—that when immersed in water they take up as much space as they occupy in the air. From this it was deductively inferred that if two bodies of equal weight are of unequal size the larger will displace more water than the smaller. It was already well known that, bulk for bulk, gold is heavier than silver. All that remained was to devise an experiment to apply the general law to the particular case. The question raised was thus answered, and the answer was assuredly true.

In the example of the painting in the cave the suggested explanation is related to known customs of the Australian aborigines.

(iii) Verification.—In considering the part played by supposition in these inquiries we see that much has to be done after the explanation is suggested. Many suppositions are sooner or later disproved, by being found to be inconsistent with some facts which were unknown, or were not taken into account, when they were formed. Those which are upheld are upheld by those same facts. In Poe's tale Dupin's solution involved a sailor and an ape, neither of which was among the facts from which he inferred their existence and action. All remained theory till confirmation by the sailor himself brought the required verification. We make many suppositions in life which turn out to be false. Nor is this confined to matters of every-day experience. The history of science is full of discarded suppositions. Indeed, it has been said that "to try wrong guesses is, with most persons, the only way to hit upon right ones." [1]

To give ground for belief, or to justify action, a supposition must be tested by comparing its consequences with other facts, of as different a kind as possible from those that suggested it. Till thus verified a supposition remains a mere suggestion indicating a line of inquiry.

6. **Application.**

When a general relation is proved to be true, it can be extended to other cases by deductive reasoning. These are thus brought under the same system of thought, and a new advance is made towards the unification of knowledge. The application of the law of specific gravity discovered by Archimedes has by no means been confined to deciding the composition of Hiero's crown.

Nor is such application tentative. The principle of gravitation is extended without doubt or hesitation to every fresh star found by the telescope, and to every minute particle of matter revealed by the microscope. The chemist is sure that

[1] Whewell: *Novum Organon Renovatum*, p. 79.

elements always combine in the exact proportions he finds illustrated in the laboratory. Thought is possible only on the assumption expressed in the Principle of Identity that the nature of things, as shown in their relations with other things, is constant.

It is in this assurance of its results that this step in gaining knowledge differs from the last. In that also results are inferred. But the inference is a question, which has to be answered by appeal to fact. The law from which the results are derived is still on its trial. Application is reached only when that trial is over, and the law established. No one doubts that its results are true, and were a fact to appear to be out of harmony with them we should strongly suspect that the fact was wrongly described or wrongly interpreted.

No sane person, for instance, would credit that an account of a boy who climbed up into the air and continued his upward progress uninterruptedly, never returning to earth, referred to anything more real than an Indian conjurer's trick. We reject without hesitation the statement that "in many places of the sea are great rocks of stone of adamant (loadstone), which of its nature draws iron to it; and therefore there pass no ships that have either bonds or nails of iron in them; and if they do, anon the rocks of adamant draw them to them, that they may never go thence."[1] That there is personal testimony both to the disappearing boy and to the rocks of adamant does not affect our incredulity. And that incredulity is based on acceptance of laws of nature with which the asserted facts are irreconcilable. We call for no rebutting testimony; we know that the statements cannot be true, because they cannot be fitted into a wide system of knowledge in which so many other facts find their meaning.

[1] *Voyages and Travels of Sir John Mandeville*, ch. 27.

7. **Summary of Steps of Inference.**

The establishment of a system of knowledge of some part or aspect of the world has, then, four steps or stages—

(1) Observing	*Induction.*
(2) Supposing	
(3) Verifying	
(4) Applying	*Deduction.*

8. **Time taken by Investigation.**

These steps may be taken in rapid succession, or may be spread over many years. Every discovery in science gives a new meaning to facts which, as mere facts, were always known to mankind. The explanation may be suggested a long time before it can be verified. Newton put aside for fifteen years his supposition that gravitation ruled the revolution of the moon, because he deduced as a consequence that it ought to be deflected from the tangent to its orbit a little over fifteen feet a minute, while its real deflection appeared to be only thirteen feet. There was a clashing of theory with observed fact. But the theoretical conclusion rested on the estimate of the distance of the moon from the earth. Error might be there, and not in the supposition. This proved to be the case, and when Newton repeated his calculations on the basis of the more accurate measurement, the apparent discrepancy disappeared. Many suppositions in science still await verification.

Evidently, till knowledge is complete, the fourth step will be ever taken anew. In this way science predicts what will happen in cases in which the conditions are exactly known. Astronomy foretells eclipses, and even the return of comets, with the greatest exactness. The reasoning is mathematical and abstract; the physical laws to be considered are few and well established. These are the conditions in which abstract reasoning is most sure. So it is that a science becomes more perfect as a system of thought, in the degree to which mathe-

matical reasoning can be used in it. At the same time, it must be remembered that this perfection is an abstract one; so that the results are exactly applicable to concrete facts only when they are modified to meet any new factors those facts may introduce.

9. Hypotheses.

Supposition is, then, the pivot of the inductive process. To it the gathering of facts is preliminary, for without it they are dead to thought. After it is made, everything that is done is done to perfect it. Every supposition is tested, rejected, or modified, as facts dictate, till at length the survivor stands sure and verified. The reasoning becomes deductive as soon as the verification is assured.

In ordinary life we speak of 'guesses' or 'suppositions' according to the amount of confidence we feel in them. In science they are all known by the corresponding Greek word *hypotheses.* So it may be said that the whole of the thought by which we attain knowledge of the world of fact is the elaboration and justification of hypotheses.

(i) CONDITIONS.—It may be asked whether there are any kinds of hypotheses we are not justified in making. Here, again, we appeal to daily life. We do not think it justifiable to make a wholly baseless assumption, as, for instance, to suppose without evidence that a person is guilty of a crime. In Poe's story such a supposition was made by the police. Though the money was found lying on the floor, they arrested the bank-clerk who had brought it, on suspicion that he had committed the murders in order to steal it. The same limitation applies in scientific reasoning. Every hypothesis should be based on the facts of the case, and be consistent with established knowledge.

In the next place, no hypothesis should be formed from which conclusions cannot be drawn. For such an one must remain for ever out of contact with facts, and so be capable

of neither proof nor disproof. It could be nothing better than a picturesque simile.

That a hypothesis rest on fact and lead to something beyond itself are the only limitations thought admits to its right to guess at explanations.

(ii) SUGGESTION BY ANALOGY.—Given certain facts, how can we make use of past experience to explain them? We can only seek for a case resembling this one as closely as possible in points material to the question. This is exemplified in the analogy which led Archimedes to solve the problem of the crown by comparison of specific gravities, and in the interpretation of the pictures in the cave of Niaux.

In dealing with our fellows we continually make such assumptions as that what pleases Jones will please Brown, because their tastes are similar. The likeness in tastes is pertinent to the question. In every other way Jones may be very unlike Brown—in size, wealth, strength, intellect. All that is irrelevant. But if each is a lover of music, and Jones was delighted with the rendering of a sonata by a certain pianist, it is a fair *working hypothesis* that we shall gratify Brown by providing him with the opportunity of hearing a similar performance. We continually direct our conduct by such working hypotheses.

In scientific investigation they equally have their place in guiding inquiry. Nobody researches at random, and even a hypothesis which is doomed to be overthrown by fresh discoveries, if only it harmonise the facts on which it is based, may do good service. For example, the long discarded Ptolemaic hypothesis enabled the ancient astronomers to calculate the motions of sun, moon, and planets, and so did much for the advance of astronomy.

(iii) FALLACIOUS USE OF HYPOTHESES.—The most serious fallacy that can be committed in induction is to take a hypothesis for an established truth. This is a mode of begging the question which impatience to reach and enounce

conclusions often leads men to commit, both in affairs of daily life and in science. A guess may be brilliant and attractive, but it is nothing but a guess until it has been through the mill of verification. Perhaps, if the term 'guess' were generally substituted for 'hypothesis,' at any rate in its inception and before evidence had begun to strengthen its probability, the fallacy would be less frequently committed.

The assumption of false axioms is now seen to be the taking what are at best working hypotheses for established truths. This, too, is the mistake made when unjustified conclusions are drawn from analogy. It is not amount of resemblance, but pertinence of resemblance, that gives probability to a hypothesis based on analogy. And it is never more than a hypothesis. Even when the analogy is a true one, it holds only in the one point of similar relation: to extend it further is fallacious. So it becomes clear why arguments from analogy are so often unconvincing.

It need hardly be pointed out that, though we all guess at all sorts of solutions of all sorts of problems, our power of guessing right is by no means uniform. Every guessing game brings that out. And making a guess and forming a hypothesis are plebeian and patrician members of the same class of mental activities.

As in small things, so in great ones, originative minds are few. In all branches of science—or exact knowledge—the many do little but receive the results of others' thinking. Their 'experiments' are not truly experiments at all, for they are not devised as means of solving a problem as yet unsolved, but are merely imitations of what others have done before. It is the exceptional mind that makes new systems.

Discovery, then, cannot be reduced to rule, for its essential feature is a question of mental endowment. No training can make a discoverer, though training can render more effective the work of one born with the discoverer's power.

To understand the reasoning by which a discovery is made

is another matter. The logic of induction is simple in its form. The difficulty is not in recognising the essential steps after they have been taken, but in taking them. The logical analysis—like all analysis—is abstract; the process is the same whatever the matter. But it can be applied only *in* the matter, and to apply it in a new way demands deep knowledge of what has been already accomplished, as well as that touch of genius of which we have spoken.

10. **Induction and Enumeration.**

Scientific induction aims at establishing laws universally true, by the analysis in thought of particular events. It rests on the assumption that relations are constant. Hence, the number of cases it examines is immaterial. If the analysis be accurate, one example is as good as a thousand. The only object of repeating observation or experiment is to ensure its exactness. One instance is enough to enable a chemist to enounce the composition of any new body he produces in his laboratory, with as much confidence as that the combination of two volumes of hydrogen with one of oxygen gives water, though the latter has been exemplified many thousands of times.

Induction does not rest on 'the counting of noses.' Even if in any case we count them all—and that is very seldom possible—we cannot thereby find a universal law of relation. That is grounded in the nature of things, and can only be laid bare by analysis of that nature. Counting, or enumeration, does nothing of this. It merely sums up the results of unexamined experience.

Consequently, the idea that scientific induction is 'imperfect' or 'incomplete,' because it does not examine all the instances, and that an exhaustive counting would be a 'perfect' or 'complete' induction, is founded in an entirely mistaken conception of the work of thought. To count, and then sum up in a single formula, is not induction at all. It

is a convenient economy of memory, but it gives no power of extending knowledge. A summary of facts can give no ground for inference outside those very facts; but, as we have seen, an induction is not established until it has passed beyond that narrow range. And it can so pass because it is not concerned with facts as independent, but with facts as embodiments of relations which are universally true; that is, as constituent elements of systems.

It follows that 'induction by simple enumeration' is wrongly named. Uncontradicted experience of uniformity suggests a hypothesis, which it is the work of scientific induction to test.

CHAPTER X.

CLASSIFICATION AND DEFINITION.

1. Development of Classification and Definition.

We pointed out, at the beginning of the last chapter, that the first step in gaining knowledge is the grouping of like things and events and the calling them by a common name, and that for the ordinary experiences of life this work has been done by our ancestors and its results embedded in language. Learning to talk is not merely a learning of words, but also a learning of their application.

At first a child applies words too widely: the baby will call every man 'dada.' But, as differences are noticed this rough use is gradually corrected. The child not only distinguishes his father from all other men, but in the indefinite class of 'men-who-are-not-daddy' he progressively marks out more distinct classes—a small class of uncles, larger classes of soldiers, policemen, sailors, clergymen, doctors, and so on, according to the types of men that enter in a determinate way into his experience.

The hearing names applied by those about him helps him, but it is only as he notes the distinctive characteristics which those names imply that he applies them correctly. A soldier or a policeman is a man made different from others, first by garb, afterwards by both garb and occupation. So a policeman becomes known as a man clothed in blue, with long coat, belt, and helmet, who walks the streets and is a terror to youthful delinquents; a soldier as a man clad in khaki, who is trained to fight for his country.

In all this is the germ of both classification and definition. For ideas of things are becoming *clear*, as those of one kind are distinguished from those of another; and *distinct*, as the characteristic marks by which this separation is made are explicitly apprehended.

In most matters of common every-day life we carry this process only as far as is demanded by our practical needs. To many inhabitants of towns, for instance, classification of such common objects as trees and birds goes little beyond the child's classification of men. They have noted but few differences, and that notice has been cursory and accidental. Nor have its results been fixed by learning specific names. The ideas of the sub-classes are indistinct, and soon merge again into that of the wider class.

Only when an interest leads us to seek fuller knowledge in some special direction do we feel the need for greater definiteness in our grouping of facts, and for the more exact insight into their nature which will make the grouping accurate. Then we study the facts with this purpose in view.

So we find in use classifications and definitions of all degrees of precision. In subjects which we have studied intensively, our classifications and definitions may be exact and accurate; in others they may be of the most primitive description; in many they will be of some intermediate worth. Here, as elsewhere, scientific knowledge differs from that of every-day life only in the stage of perfection reached.

2. Classification.

(i) General Characteristics.—Let us now consider the nature of classification.

(*a*) *A Mental Arrangement.*—In the first place, it is a mental act. It gathers things together in thought on the ground of their known resemblances.

At times we may afterwards arrange them in space on the plan we have laid down, just as, in other cases, we may

express our thought in writing. But the planning is the classification, which the arrangement simply copies. For instance, as a matter of practical convenience, I may arrange the books on my shelves according to the subjects of which they treat. But the discrimination of subjects, and their relations to each other, had first to be settled in my mind. The placing them on the shelves only carries out the grouping already made in thought.

But in by far the more numerous cases such a grouping in space of the actual things is out of the question. The botanist classifies plants. Obviously this does not imply that he first collects all plants together, and then separates them into groups, planting the oaks in one field, the roses in another, and so on. He deals with his knowledge of plants, and holds the various classes apart in thought.

Actual grouping of things, then, is not a part of classification. When it is done, the objects dealt with are few, and their segregation is merely an expression of the classification, not the classification itself.

Dissection.—Neglect to keep clearly in mind that classification is a mental grouping leads to the production of spurious 'classifications.' Probably few would fall into the mistake of supposing that an enumeration of the parts of a plant is a classification of plants, or that counting the squares on a chess-board is a classification of chess-boards. Yet, just such a confusion between classification and dissection, or physical partition of a thing into its parts, appears in many school books on grammar. They 'classify' sentences into simple, compound, complex, and subordinate. Nevertheless, when they treat of the last named they rightly describe it as a part of a complex sentence.

Conceptual Analysis.—Less crude is the error of confusing the distinction of the qualities of a thing with classification of things. In a flower we can note the colour, the scent, the size, and many other qualities. We can, if we wish, take

any one of these as the ground of a classification of flowers. But to enumerate the qualities of a thing is to describe it, not to group it with other objects.

Both these kinds of sham 'classification' break down when we apply the test of whether we can predicate the name of the wider class of each of the sub-classes. We can say 'All roses are flowers,' but to affirm that 'A pistil is a flower' or 'Red is a flower' or 'A subordinate clause is a sentence' is to utter nonsense.

(*b*) *Determined by Purpose.*—To decide obligation to military service men are first classed by age. Then those liable for service are further classified according to physical capacity. But in civil life an altogether different grouping, based on occupation, prevails.

This is an example of the general truth that we never classify at random, but always with reference to some purpose.

If the question be the setting out of ornamental flower beds, a classification of flowers according to appearance will be pertinent. If it be of when to plant, and when to take out of the ground, quite another grouping is needed. For each purpose the other classification would be useless. Nor would either be of value in an inquiry into medical properties of plants, or help to decide what plants should be planted in different kinds of soil.

(*c*) *Based on Definite Principle.*—It follows from the fact that the same objects can be classified in various ways according to the purposes with which the classifications are made, that every classification should be based on a principle related to that purpose.

If in examining a number of schoolboy essays the object be to compare their literary promise, the marking—or classing—will be determined by questions of such qualities as style, arrangement, and imaginative power. If it be to test spelling, the number of departures from orthodox

orthography will be decisive. Other classifications might equally well be made on such bases as handwriting, grammatical correctness, or amount produced in a given time. Each would yield a good classification; that is to say, one 'good' for the purpose in view. For any of the other purposes it would be not only useless but misleading.

Fallacy of Cross Division.—Yet, to confuse the principle of one classification with that of another is a common error. Then a wider class is divided into narrower classes which are not altogether separate from each other, and which together may, or may not, include the whole of the wider class.

If I classified pens into fine-nibbed, broad-nibbed, steel, and gold, pens, I should doubtless mention four sub-classes of pens. But I should not include all pens, and some would belong to more than one group.

If successive steps in classification be made, there is no objection to taking a different basis for each, as in the case of men for military service. The classification is vitiated only when more than one principle is applied in a single step of division. It will not do to classify goods as bulky, light, beautiful, and 'made in Germany.'

Such errors are called *Fallacies of Cross Division.*

(*d*) *Classifies a Definite Whole.*—Classification is always of facts of which we have enough knowledge to see that they are related to the purpose with which we make it. In classifying, as in other modes of thinking, we keep our thoughts within the limits of a universe of discourse,[1] here known as the *Totum Divisum*, or Whole to be Divided.

It may be a very small one. If I classify my books I deal with a 'universe' limited to the books I possess, and they are but an infinitesimal part of the totality of books in the world. Others might be useful to me, but they are unavailable. For the carrying out of the purpose to do a

[1] See pp. 47-48.

definite piece of work my own books are all that are pertinent.

In another instance it may be that everything called by the common name is pertinent to the purpose. If we wish to investigate the nature of plants we need a classification embracing all plants.

In either case it is evident that we must have enough knowledge of the whole we wish to classify to mark it off from other things, with which we do not intend to deal. So we must know the chief characteristics which distinguish the things we include from those we exclude. To express this knowledge is to give a definition. It may not be scientifically satisfactory, but it must be sufficiently accurate and definite to work with.

In ordinary matters this is not difficult. Language is an adequate guide. But there are cases in which the boundaries of the class we propose to divide are not plainly marked. For example, sponges and sea-anemones were long regarded as belonging to the general class of plants. This affected classification both of plants, among which such forms of organised life were wrongly included, and of animals, from which they were mistakenly excluded.

(*e*) *Orderly.*—If we examine the classifications embodied in language, we find that they are not limited to one step. The example of men for military service illustrates how the sub-classes obtained by the first division are themselves divided into smaller classes, on the basis of a quality common to some of the members, but not to all of them. In such a *Progressive Classification* it is essential that at each step the sum of the sub-classes should include every member of the widest class with which the process begins. This is secured if each step be correctly taken.

If the characteristic chosen as the principle of division be not one which in some form is found in every member of that class, a preliminary class must be made of those

which do not possess it, or they will be omitted altogether. If we began by dividing ships on the basis of their rig, and considered only those that have some kind of rig, we should omit, not only all air-ships and submarines, but the great majority of steamers. For rig is a characteristic of sailing-ships. Consequently, variation of rig is an appropriate principle on which to classify them. But it is not suitable as a ground on which to begin a classification of *all* ships.[1]

So we get the rule that continued classification must proceed step by step (*Divisio non faciat saltum*)—that each set of co-ordinate species must include the whole of the genus from which they are formed. Then we secure that no intermediate classes are passed over.

(ii) FORMAL DIVISION.

(*a*) *Division by Dichotomy.*—As all classification is a sorting of certain selected things according to the relation of their qualities to the purpose in view, it must be based on examination of these qualities. This is its material factor. But the purpose may be satisfied by determining merely whether the chosen quality be present or absent, remembering, of course, that absence means presence of an incompatible quality of the same general kind.[2]

For example, if mirrors are to be of much use to soldiers on active service they must resemble glass in reflective power, but not in fragility. So from the wide class of materials capable of receiving a high surface polish we must exclude the sub-class of brittle things. We then take the remaining class of things-that-are-not-brittle, and divide it on the basis of durability of polish into those which resist rust and those which do not. This excludes such a metal as steel, because it is liable to rust when exposed to damp. And so on, each step rejecting, on some new ground, some

[1] Cf. p. 56. [2] See pp. 46-47.

materials as unsuitable for mirrors to be used on service, till it may be decided that silver best meets the conditions, and that the mirrors should be made of that metal, or faced with it.

At each step of such a progressive classification the simple presence or absence of one quality is taken as the basis of division. So the earlier steps are—

Materials with reflective power

non-brittle | brittle

non-rusting | rusting

Such a process is known as *Division by Dichotomy*, because at each step the sub-classes are formed on the ground that one possesses, and the other does not possess, a particular quality. There are, thus, two, and only two, sub-classes at each step.

It is useful when we wish to reach a definite character by the elimination of alternatives. Were we to consider a number of these at once, confusion would probably be introduced. Whether the classes through which the process is successively effected are marked by a name positive or negative in form does not affect its character.

In such a division we secure at each step that the class we divide is wholly exhausted by the two sub-classes, and that these are mutually exclusive. We have, therefore, a formal assurance that the division is correct in these essential points. For this reason this was the only kind of division admitted into formal logic.

(*b*) *Tree of Porphyry.*—For over sixteen centuries the typical example was known as the 'Tree of Porphyry.' This was an attempt at classifying all existence, and was, there-

fore, related to the most general purpose of knowledge. It was—

Summum Genus..........................Substance

Differentia.................Corporeal Incorporeal

Subaltern Genus .... Body

Differentia.................Animate Inanimate

Subaltern Genus .... Living Being

Differentia.................Sensitive Non-sensitive

Subaltern Genus......................... Animal

Differentia....... ....Rational Non-rational

Infima Species .... Man

Socrates, Plato, and all other Individuals

(*c*) *Technical Terms.*—In connexion with such a scheme was developed the terminology of classification.

The widest class of all (substance) is the *Summum Genus* (widest genus), the narrowest (man) is the *Infima Species* (lowest species), which is reached whenever the next step in division yields not classes but individuals.

The direct series of classes between these two is a *Linea Predicamentalis* (predicamental line). Each intermediate class along this line is at once genus to those below it, and species to those above. Each is a *Subaltern Genus* or a

Subaltern Species, according as we pass down or up the line. In relation to each other a genus and the species into which it is immediately divided are *Proximate*.

The attribute on the presence or absence of which each step of division is based (the *Fundamentum Divisionis*) is the *Differentia* (differentiating mark or 'difference') by which two co-ordinate species are distinguished from each other.

(*d*) *Value of Dichotomy*.—A series like that here worked out with the positive terms could be developed from each negative term. Thus, many predicamental lines start from the same *summum genus*. For every term reached is formally divisible into species which possess, and species which do not possess, a chosen quality.

Such a process involves only so much reference to fact as is needed to determine the successive principles of division. But, owing to its formal guarantee of completeness, a division by dichotomy is at times useful to test a classification made on other lines.

When the ground of division is not the presence or absence of a quality, but its presence in a variety of degrees or forms, dichotomy obscures the true relation of the sub-classes. If we classify triangles on the basis of the relative lengths of their sides we get immediately three co-ordinate species, each of which is a proximate species to the genus triangle. To make a first step by separating equilateral triangles from those that are not equilateral, then to divide the latter into isosceles and those not isosceles, and finally to find that the remainder is an *infima species*, would obscure this, and, to that extent, vitiate our system of thought about triangles.

The number of proximate species to any genus should, then, be decided by examination of the facts.

(iii) Principles of Classification.—We are now in a position to set forth the conditions which a good classification should fulfil:—

(1) Each act of division must be on a single basis.

(2) The sum of the species must be co-extensive with the genus.

(3) Everything outside the genus must be excluded.

(4) Each step must be proximate to the one preceding it.

(iv) Kinds of Classification.

(*a*) *Special and General Classifications.*—We are constantly classifying things and events for all sorts of purposes, and, provided the classification is based on a suitable principle and fulfils the conditions just set forth, it is a good one for that purpose.

On the other hand, the purpose may be to systematise knowledge as a whole. Then the classification seeks to arrange the things with which it deals as they are related in the general scheme of the universe. The result is not the private concern of this or that person or body of persons; it is a systematisation of the total knowledge of mankind of that form of existence, and is built up, tested, and rectified by the co-operative work of many inquirers.

Such a classification is often called *Natural*, and all special classifications *Artificial*; but these names are unfortunate.

All classifications are based on real characteristics of things, so in that way all are 'natural.' All are 'natural,' too, in the sense that each special classification, well adapted to its purpose, is the one it is 'natural' for us to make in relation to that purpose. A classification of plants according to their curative properties is 'natural' for the purposes of medicine, and in relation to that purpose the general, or botanical, classification would be 'unnatural.'

On the other hand, every classification is made by our thought. It corresponds to no physical arrangement in space; it is only a way in which we choose to think things together for purposes of our own. In this sense, every classification is 'artificial.'

But though the terms are inappropriate, the distinction they are intended to mark is a very real one. It is that between classifications for special practical purposes and classification for the general theoretical purpose of knowledge.

A special classification is made in the interests of some practical end, and is based on knowledge of the relations of things to that end; as, for example, the classification of materials with reference to their suitability for making mirrors.

An instance of greater permanent value is the classification of plants made in the eighteenth century by Linnæus. "Before his time botanists had only given one name to a set of plants; calling all roses, for example, by the name *Rosa*, and then adding a description to show which particular kind of rose was meant. Thus, for instance, for the Dog-rose they were obliged to say . . . 'common rose of the woods with a flesh-coloured sweet-scented flower.' . . . Linnæus was the first . . . to give a *specific* . . . name to each particular kind of plant, describing the plant at the same time so accurately that any one who found it could decide at once to what species it belonged. To accomplish this he classified all plants, chiefly according to the number and arrangement of their stamens and pistils (or the pollen-bearing and seed-bearing parts), and then he subdivided them by the character and position of their leaves and other parts." [1]

The aim is practical, but the classification brings together species which in all except the number of stamens and pistils are far apart, and, because of disagreement in that point, separates species which are nearly related by affinity. A general classification of plants or animals is not based on some arbitrarily chosen external mark, but on consideration of their whole nature. Though the species of to-day may have had a

[1] Miss A. B. Buckley: *A Short History of Natural Science*, pp. 207-208.

common origin in the remote past, yet each shows a unity of structure and mode of life which are transmitted to every new member of the class. Classification deals with things as they are. It draws a map of that section of evolution in time in which we ourselves find a place.

Thus, a general classification is possible. Nowhere is it fully accomplished. In some cases much remains to be done. For instance, "the number of German species of hawk-weed has been fixed by one author at 300, by another at 106, by a third at 52, while a fourth is content with only 20." [1]

(*b*) *Subsumptive Classification.*—When, instead of making each step of a progressive classification on an independent basis, the marks chosen are related to one general principle, the classification is subsumptive. In the biological sciences the principle is that of descent. So the classification becomes a genealogical tree.

The aim of such a classification is so to systematise the things with which it deals that those which resemble each other in important points shall be grouped together, and that, in the subordination of classes, the nearness of groups to each other shall be in proportion to the amount of such resemblance between them.

(v) CLASSIFICATORY AND EXPERIMENTAL SCIENCES.—Classification setting forth relationships of descent belongs to the biological sciences.

In these, classification plays a larger part than in the physical sciences which investigate inorganic nature. There the chief aim is to establish relations of causation—to show under what conditions one event inevitably produces another. The things with which they deal are, in comparison with organic beings, simple in structure, and easily classified. They can also be subjected to all kinds of experiments. So these are called the 'experimental sciences.'

[1] Joyce: *Principles of Logic*, p. 388.

The complexity introduced by life and development in time restricts the application of experiment to organic beings, though much work is being done in the investigation of transmission of qualities. So, though the biological sciences aim ultimately at the establishment of causal relations, they are far less advanced towards that goal than are physics and chemistry. They are, in consequence called 'classificatory sciences,' but this must not be taken as implying that with the attainment of perfect classification and connected definitions their task would be accomplished.

Geology is intermediate between the two stages. It deals with inorganic matter, but it cannot experiment on the formation of the earth's crust in past ages.

Next, it should be noted that scientific classification does not stop with grouping the objects of experience. Chemistry analyses these into their elements, and regards the classification of elements as fundamental. That of compound substances is based on their composition. Similarly, biology aims at classifying the original forms of organic life, and at making that the foundation of the classification of present species. In each case, the process is inductive, proceeding by hypothesis and verification of hypothesis.

(vi) Nomenclature and Terminology.—Names in common use have not the precision in reference and meaning which scientific thought requires. The disadvantage of using them is seen in the sciences which deal with human nature and actions. Such words as freedom, happiness, virtue, nature, pleasure, wealth, have wide ranges of meaning. A writer is apt to pass unconsciously from one to another; a reader to interpret in one sense what is meant in another. So, instead of that steady co-operation which has done so much for the advance of the physical sciences, there have been constant misunderstanding and controversy.

The physical sciences have avoided this by evolving more or less perfect nomenclatures consisting of technical terms

which often contain their meanings in themselves for those who know the languages—generally Greek and Latin—from which they are formed. These terms may exist side by side with those of common speech; as, for instance, the names of plants or birds. When, as in chemistry, they denote mainly the products of the laboratory, they often have no such vulgar competitors.

The advantages of such scientific nomenclatures are obvious. Not only are the names unambiguous, for they carry no associations of every-day life, but they are current among men of all nations and tongues.

Similarly, these sciences have special *Terminologies*, or collections of terms, naming qualities and parts of objects, such as the calyx, corolla, stamens, and pistils of a flower. By use of these, descriptions are made precise. As might be expected, nomenclature and terminology are particularly important in the classificatory sciences.

The great safeguard of technical terms against ambiguity is immunity from the attrition of every-day use. This safeguard their generally unattractive form does much to maintain.

3. **Classification and Generalisation.**

In classifying we make a series of acts of division, in each of which we take some general quality, that appears in different forms in the things with which we are dealing. These forms we make the differentiating characteristics of the various species. By adding them to the connotation of the name of the genus we get the connotations of those species.

If we follow the process in the other direction we pass through a succession of acts of generalisation. By omitting the distinguishing characteristics we gather the species into the genus, the connotation of which, therefore, contains less than that of any of the species.[1]

[1] Cf. pp. 56-57.

Classification and generalisation, then, are reverse, though related, processes. The one passes downwards, segregating on the basis of differences; the other upwards, aggregating on the ground of likeness. Neither is possible unless both likeness and unlikeness can be found.

How far either is carried depends on the purpose in view. Botany does not need to pass upwards beyond plants, nor zoology beyond animals; but biology must regard these as sub-classes of the wider class of organic beings.

So with the lower limit. In drawing up a catalogue of books for the general reader, it may be enough to reach such classes as are marked by the names of the great departments of knowledge—history, geography, geology, fiction, and the like. But for the specialist student in any one branch of knowledge, that may be the widest pertinent class, which must be grouped in a number of successive steps before he has a key sufficiently definite to serve his purpose of finding what books are available on any particular matter in which he may wish to carry out research.

4. Definition.

(i) Connexion with Classification.—Classification and definition involve each other. The one systematises the denotation, the other the connotation, of the same sets of things.

In classification we first determine our *summum genus*. This implies that we know what it is. The more accurate this knowledge, the more secure are we against dealing with more or less than we should. To name the attributes which at once unite all the members of a genus and mark it off from all other things, is to state its connotation; that is, to define it. As the etymology of the word suggests, 'to define' is primarily to set limits—to include what should be inside, and to exclude what should be outside.

Each species differs from all its co-ordinate species by

possessing some quality of the genus in a specific form. This is its *differentia*. To add this to the name of the genus gives an adequate definition of the species. There is no need to state the definition of the genus, because the very act of classifying implies that this is known. So we get a very simple rule for definition which shows its close connexion with classification: *The definition of a species is the sum of proximate genus and differentia.*

(ii) PRINCIPLES OF DEFINITION.—The more clearly a definition is stated the better. All figurative or metaphorical language is out of place. 'Anger is a torrent that sweeps away the frail barriers of prudence' may suggest a vivid image of the difficulty of checking a burst of anger, but does not at all tell us what anger is, and so is not a definition.

Nor are tautologies. That 'mutability is inconstancy' may convey knowledge as to the meaning of 'mutability' to one to whom that word is unfamiliar, but who knows the meaning of 'inconstancy.' But it does nothing more. The statement is purely verbal. If, however, the rule to define by adding to the genus the differentiating characteristic of the species be followed, such errors cannot be made.

We do not have tautology when the name of the genus is also part of the name of the species, as, for instance, 'equilateral triangle.' Then the *differentia* unfolds the meaning of 'equilateral.' So the whole name of the species, consisting of the two words 'equilateral triangle,' is defined. That the name of the genus is part of the name of the species is accidental and nothing to the point.

As definition states the special characteristics of a class of things it can only be clearly expressed in positive form. Euclid's definitions of a point as 'that which has no parts and which has no magnitude,' and of parallel straight lines as 'lines in the same plane which never meet,' are negative. But each really assumes a positive quality. To make good definitions these should be expressed. Only when we are

dealing with a negative idea, such as 'inequality,' is it permissible to express the definition negatively, as 'absence of equality'; and this is meaningless apart from the positive definition of 'equality.'

A good definition, therefore, *gives the connotation in clear and positive terms.*

(iii) KINDS OF DEFINITIONS.

(*a*) *Special and General Definitions.*—The close relation of definition to classification involves that there are special and general definitions, connected with the corresponding kinds of classification.

General definitions would be as irrelevant to special purposes as would general classifications. For they set forth the characteristics important in determining the place of the class defined in the general system of the universe, and those are probably not the most important for a limited practical purpose. The botanical definition of a plant would not be serviceable to a market gardener, nor would it express *his* knowledge. Nor would the zoological definition of 'sheep' state what it is important for a shepherd to know.

This throws further light on what we found in an earlier chapter, that though we all use words with the same reference, we do not all have the same knowledge in our minds when we use them.[1] The experience of each one of us has made us more or less familiar with certain relations of things to ourselves, and these loom large with us. But the experiences differ in every possible way. A boy would not define a cricket-ball in the same way as would the maker, and neither definition would satisfy the requirements of geometry.

We will exemplify special definition from the same Linnæan system to which we referred in classification. "In describing the geranium, for example, [Linnæus] mentions first the 'sepals,' or little green leaves under the flower; he says they

[1] See pp. 19-22.

are five, and very pointed; then the 'petals,' or flower-leaves, are five also, growing on the sepals and heart-shaped; the 'stamens' are ten in number, and grow separate; the little vessels on the top of the stamens, which are called 'anthers,' and hold the yellow dust, are oblong; the 'pistil,' or seed-vessel, is formed of five parts, which are joined together into one long beak ending in five points; the seeds are covered with a skin and are shaped like a kidney, having often a long tip which is rolled round in a spiral (like a corkscrew).

Here we have a definition of the *genus* geranium; but many geraniums will answer to this description, so he goes on to describe some more special characters. The sepals in this particular specimen, he says, are joined together in one piece; the stem of the plant is woody, the joints are fleshy, the leaves are slightly feathered at the edge. These last characters are peculiar to this kind of geranium, which he calls *Geranium gibbosum*, and here we have the *specific* name. Any geranium which has the woody stem, the joined sepals, the fleshy joints, and the feathery-edged leaves, will be the species called by Linnæus *gibbosum*."[1]

General definition, like the classification with which it is connected, is inspired, not by consideration of our changing practical needs, but by the theoretical purpose of systematising knowledge. It is this general definition which is referred to when we speak of 'the' definition, or 'the' connotation, of a name. Then the reference is to the general system of knowledge, and not to any particular, and it may be very limited, application of part of that system.

From this general standpoint, all special definitions are descriptions; they may contain none of the general connotation. Both the proximate genus to which they refer the species, and the differentiating characteristic by which they mark it off from its co-ordinate species, may be im-

[1] Miss A. B. Buckley: *Op. cit.*, pp. 208-209.

material from the point of view of knowledge of what a thing is in itself.

(*b*) *Explicative and Genetic Definitions.*—The exposition of definition by genus and difference sets forth the characteristics of a class as they are, without regard to how they came to be. The change of view brought about by evolution affected definition as well as classification. In biology, the general definition seeks now to express affinity. So the characteristics by which it marks the genus, and the distinguishing marks of each species, are those which best show descent. The subsumptive classifications and the definitions related to them together exhibit the biologist's knowledge of evolution. The definitions aim at setting forth, not only what a class is, but how it came to be what it is. A definition of this kind is called *Genetic*; that is, indicative of origin.

In the mathematical and physical sciences also this form of definition has in many cases superseded the older definitions. For example, a circle was defined by Euclid from consideration of the completed figure. Modern geometry would rather define it as the figure marked out by the revolution of a straight line in a plane round one of its ends which is fixed.

These changes of view do not affect the logic of classification and definition. The question of what shall be taken as the guiding principle is one for each special science. Logic only considers how the chosen principle is applied.

(iv) Inductive Nature of Defining.—That definition results from induction, and proceeds by way of forming a hypothesis and then verifying it by appeal to facts, is apparent. For, that the qualities chosen are those characteristic of the whole class, and, as a whole, possessed by nothing which is not a member of that class, is at first a guess into the nature of things. It is suggested by earlier knowledge, tested and verified by advancing knowledge. So it is that definitions are often modified as knowledge becomes fuller and more exact.

(v) Value of Definition.—Definition of any kind is but one way out of many of expressing meaning. No definition exhausts the richness of the nature of any class. From many points of view, a shepherd's knowledge of sheep is far richer than that of a zoologist, and for the care and nurture of sheep is much more to the point. If we said that definition expresses meaning, and then restricted definition to the general definition of zoology, we should be taking the absurd position that the shepherd's probable inability to give such a definition, and very likely to understand it if anyone told it to him, is evidence that he has no real knowledge of sheep after all.

It is very easy to exaggerate the practical importance of general definitions as mental illuminants. Definitions offered to ignorance do little or nothing to lessen that ignorance.

As we study a class of facts the power to define grows, and we can frame a satisfactory definition only when we have learnt much of attributes and relations. If we seek to gain knowledge from another of some class of unfamiliar objects, say 'Tanks,' we need as full a description as possible. To be told its place in a scheme of classification, probably quite unfamiliar, helps us not at all. And it is this fixing of place in classification which is the essential work of definition.

Traditional logic regarded only general classifications and definitions, probably on the ground that logic is concerned with general principles of thinking which lead to the attainment of theoretical knowledge. But practical knowledge is as truly knowledge as is theoretical knowledge, and its attainment occupies no small part of our energies. The frequency and the value of special classifications and definitions ought to be recognised. They should fulfil the same conditions as general classifications and definitions. The difference is not in the logical principles, but in the mode of their application.

(vi) Fallacies in Definition.—Wrong application of a name involves a fallacious use of definition. Something which wants the qualities connoted by the name is labelled with it, and inferences are drawn from this wrongly assumed identity. We will give an illustration from Junius:—

"Any man who takes the trouble of perusing the journals of the House of Commons will soon be convinced that very little, if any regard at all, ought to be paid to the resolutions of one branch of the legislature, declaratory of the law of the land, or even of what they call the law of parliament. It will appear that these resolutions have no one of the properties by which, in this country particularly, *law* is distinguished from mere *will* and *pleasure*; but that, on the contrary, they bear every mark of a power arbitrarily assumed and capriciously applied:—that they are usually made in times of contest, and to serve some unworthy purpose of passion or party;—that the law is seldom declared until *after* the fact by which it is supposed to be violated;—that legislation and jurisdiction are united in the same persons, and exercised at the same moment;—and that a court, from which there is no appeal, assumes an *original* jurisdiction in a criminal case;—in short, to collect a thousand absurdities into one mass, 'we have a law, which cannot be known because it is *ex post facto*, the party is both legislator and judge, and the jurisdiction is without appeal.' Well might the judges say, 'The law of parliament is above us.'"[1]

5. The Predicables.

It is evident that every species and every individual possesses many qualities which do not enter into the connotation of its name, as fixed by its definition. This is true, whether the definition is connected with the general classification or with one made for a special practical purpose.

[1] *Let.* 48.

From the standpoint of the classification and definition adopted, can these be distinguished as of different degrees of importance for the purpose in hand? The search for an answer to this question led to the formation of a scheme of *Predicables*, that is, kinds of predications that can be made of a subject.

(i) GENUS AND SPECIES.—If the subject be an individual it can be brought directly under a *species*, and so given its place in a hierarchy of classes.

If it be a species it can similarly be placed under a *genus*.

(ii) DIFFERENTIA.—Further, we have seen that each species is distinguished from the other species of the same proximate genus by a definite *differentia* or *difference*. This difference, then, can also be predicated of the species. 'Deliberate' as well as 'false statement' can be asserted of 'a lie.'

(iii) PROPRIUM AND ACCIDENS.—The remaining attributes, often indefinitely large in number, are divided into two classes, according to whether they appear to be closely bound up with the basic qualities already examined or not. The former are called *Propria* or *Properties*, the latter *Accidental qualities* or *Accidentia*.

The theorems in Euclid's 'Elements of Geometry' are deductions of 'properties' from the combination of the definitions with mathematical axioms. That the area of a triangle is obtained by multiplying the units of length in the base by those in the height and dividing the product by two can be deduced from the theorem that the area of a triangle is half that of a parallelogram on the same base, and is, therefore, a property of triangle. But the size of a triangle is an accidental quality.

Similarly, the power to learn a foreign language is a property of all normal human beings, for it is a consequence of the nature of their minds; whether individuals do, or do not, learn any such languages is an accident,

because it depends not on this nature but on the circumstances of their lives.

When an accident belongs to every member of a species it is called an *Inseparable Accident.* The colours of many animals, such as the crow, the herring, the tiger, are examples. When, like the learning a foreign language, or the colour of other species of animals, such as horses, dogs, cats, and parrots, it belongs to some, but not to all, the members of a species, it is called a *Separable Accident.*

A distinction between inseparable and separable accidents of an individual has also been made. The former are those that belong to him permanently, the latter, such as his acts or his clothing, those that vary from time to time. This distinction is of no importance.

The scheme was drawn out in reference to general classifications and definitions, and was illustrated by the Tree of Porphyry.[1] But it may be applied also to special classifications and definitions.

As any single attribute may be taken as the basis of a new act of division, it is obvious that a property, or even an accident, in the general classification, may be made a difference in a special classification. The rank of any attribute in the scheme depends on its apparent pertinence to the purpose with which the classification is made.

6. **Examples.**

We will give a few examples of classifications, and the definitions connected with them.

(*a*) We may divide statements on the ground of their relation to fact into true and false. Each species may then be made a genus and divided on the ground of intention: false statements are separated into falsehoods, or those intentionally false, and falsities, or those unintentionally

[1] See pp. 175-176.

false; true statements into truthful, or those intentionally true, and the accidentally—*i.e.* unintentionally—true. The classification may be set out in tabular form—

- Statements
  - True statements
    - Truthful (intentional)
    - Accidentally true (unintentional)
  - False statements
    - Falsehoods (intentional)
    - Falsities (unintentional)

In accordance with this a truthful statement is defined as a statement intentionally true; and an accidentally true statement as one which is true in fact, though believed false by him who makes it.

The reader should similarly define each of the other terms in the table.

(*b*) The traditional Greek classification of the 'cardinal virtues' was based on the kind of habit involved. Thus—

- Cardinal Virtues
  - Wisdom (h. of thought)
  - Justice (h. of will)
  - Temperance (h. of restraint of appetite)
  - Courage (h. of ignoring danger)

Wisdom would be defined as the habit of thinking rightly as to conduct, Temperance as the habit of subordinating appetite to reason.

The reader should define 'Justice' and 'Courage' in a similar way.

(*c*) A classification of acts of killing, with the basis on which each step of division is made, may be thus set out—

- Killing (taking life of another)
  - Accident (unintentional)
    - Culpable (due to marked carelessness)
    - Not-culpable (not due to marked carelessness)
  - Intentional
    - Murder (malicious)
      - planned (deliberate)
      - not-planned (passionate)
    - Homicide (not malicious)
      - justifiable (in defence of life)
      - unjustifiable (in defence of property, etc.)

The definition of each species may be obtained by combining the proximate genus with the difference given in the table. For example, Justifiable homicide is intentional killing without malice in defence of life. The connotations of the subaltern genera have to be given because there are no specific terms—at any rate in common use—to denote them.

The reader should make similar definitions of each term in the table.

(*d*) Aristotle's well known classification of forms of government is made in two steps of division. The first basis is the consideration of who holds supreme power. This yields three species, according as that power is vested in one, in a few, or in the many. Each of these is then subdivided on the ground of whether the government is carried on in the interest of the ruler or in that of the country at large.

The latter are normal forms, the former are perversions Setting this forth in a table we have—

- Governments
  - Vested in one
    - Kingship (normal)
    - Tyranny (perversion)
  - Vested in few
    - Aristocracy (normal)
    - Oligarchy (perversion)
  - Vested in the many
    - Polity, or gov. by military caste (normal)
    - Democracy (perversion)

As an example of definition we say : Aristocracy is government by a few of the citizens aiming at the good of the whole community.

The reader should make similar definitions of the other terms.

CHAPTER XI.

INVESTIGATION OF CAUSAL RELATIONS.

1. Practical and Theoretical Knowledge.

Inductive inquiry aims at systematising our thought about things and events. The first step leads to classifications and definitions; the second to the establishment of laws of dependence of events upon each other. After describing the qualities of things and how those qualities are related, we proceed to ask what consequences follow from those qualities and relations. Not that the two steps are separated from each other. All fresh light on what things are shows the way to the investigation of how they act in various relations. Nor are we satisfied with mere description; we demand explanation. We want not simply to answer the questions—What? and How? but to solve the problem—Why? Interpretation presses hard on the heels of description.

Here again, science simply develops the knowledge gained in every-day experience. Thought has not to start with a mass of facts, of the relations of which it is entirely ignorant. The most elementary demands of life compel us to recognise the operation of constant laws of nature. "Even an ape must distinguish poisonous from wholesome food. Beliefs as to physical facts require to be made articulate and distinct; but we have only to recognise as logical principles the laws of

nature which we have unconsciously obeyed and illustrated—to formulate dynamics long after we have applied the science in throwing stones or using bows and arrows." [1]

Theoretical knowledge begins by making explicit what is implicitly assumed in practical knowledge. That done, it may expand that knowledge. The same factors are involved; but in the one case we are not conscious of first making, and then verifying, suppositions; in the other we are.

When a boy throws a stone he assumes that he can so communicate energy to it that the exertion of a certain force in the movement of his arm will ensure the movement of the stone with a certain velocity; that a certain direction can be given to that motion by a certain form of the arm's movement. If he hits the mark, he verifies those assumptions; if he fails, he proves them to be to some extent mistaken. As by practice he acquires skill, he makes his suppositions more consonant with facts. His purpose to hit the stone has not become clearer, but he has learnt better how to adapt means to accomplish it. His practical hypothesis that the forces at his command can be combined in a certain way becomes more precise and accurate. But he does not think out that hypothesis apart from the act.

If a man sets himself to invent a gun he has a very similar problem to solve. He, too, has to determine what force must be applied to the projectile, and in what way it should be applied, in order that the shot may execute a desired trajectory. The same laws of physics have to be applied in both cases. But in the latter they must be clearly apprehended as abstract relations, and then it has to be calculated how they should be combined.

This brings out the superiority, even for practical purposes, of theoretical over practical knowledge. In the latter the knowledge is bound up in the power. The man who does a

[1] Sir L. Stephen: *The English Utilitarians*, vol. i., p. 5.

thing skilfully is often unable to explain how he does it. A good bowler can *show* how he bowls the ball that is the despair of the batsman, but probably he cannot set out the exact combination of physical forces that makes it unplayable. Indeed, he cannot ensure it in any one act of bowling. If he could, he would take a wicket with every ball, unless the batsman were able to oppose with equal surety a counteracting system of forces.

But the inventor can lay down so explicitly the way in which the cannon should be made, and the charge inserted, that workmen who do not know the theory, and probably would be incapable of understanding an exposition of it, can make the cannon, and a gunner similarly ignorant can fire it, yet the result calculated by the inventor is inevitably secured. Further, the laws applied to the cannon are those which, in other combinations, account for every other instance of the motion of solid bodies.

Theoretical knowledge, then, is general; practical knowledge is particular.

Theoretical knowledge thus can increase understanding of events by bringing under a few constant laws cases differing from each other in many striking ways. It interprets, or explains; while practical knowledge simply describes by example, that is, by action. In the latter, thought remains implicit; its recognition of the relation of means to end is blind. In the former, both end and means are recognised with clear consciousness of their nature and their relation to each other.

The widest of all physical generalisations is the theory of the conservation of energy—that the amount of physical energy in the universe is constant, so that energy can be neither created nor destroyed. Every instance of causation then becomes simply a transfer of energy. In mechanical cases, as when one billiard-ball strikes another, this is comparatively easy to see, but when the energy changes its

form of manifestation it is less apparent. It involves the assumption of 'potential,' as distinct from 'kinetic,' or manifested, energy.

Darwin's hypothesis of 'natural selection'—or the elimination of individuals ill-adapted to their environment, and the 'survival of the fittest' to live in such environment—offered an equally wide generalisation to biology. Later students of heredity believe it to be inadequate to explain all the facts, and several hypotheses of heredity have been propounded. In dealing with living beings it is difficult to resort to decisive experiments, and no thorough agreement has yet been reached.

2. **Study of Causal Relations.**

(i) IMPORTANCE OF THE STUDY.—Though such a wide-reaching theory as that of the conservation of energy gathers up into itself all forms of physical causation, the actual causal connexions of events must be studied. Each may be accepted as a transfer of existing energy, but we want to know how this transfer is made. Between the primary fact of experience and the all-embracing abstract theory are many ranges of generality. Just as between such an idea as being-in-general, and the dog, or stick, or bread, of every-day life, we establish a whole hierarchy of classes and sub-classes; so between the idea that all that happens is a change in the working of a constant energy, and the breaking of a cup, the explosion of a mine, or the formation of a rainbow, we must establish a hierarchy of causal relations. Thus was the general theory reached, and only thus can it be grasped in thought as anything more than an empty formula. Only so, too, can we gain knowledge, either practical or theoretical, of the world in which we live.

Into the origin of causation in the universe there is no need to enter. We are investigating thought, and thought assumes causation from the first. Some have tried to

explain it away in words, but none has attempted to eliminate it from his actions. The most sceptical person drinks to quench his thirst, eats to satisfy his hunger, and stretches forth his hand to grasp his food or his drink, without doubting that his action will be successful. The question before us is *how* causal relations are found.

(ii) Analysis of Event.—In common speech we speak of 'cause' and 'effect' with various degrees of looseness. We say that people die from many causes. Some are slain by disease, some shot, some poisoned, some hanged, some crushed by motor cars, some burnt, and so on. Evidently, no constant causal relation can be established from this. The only thing common to all cases of death is preceding life; and, though we agree that life always ends in death, it would be an abuse of language to call life the cause of death.

The need for further analysis is obvious. 'Death' is a single term, covering a large number of elements which differ from case to case. Before giving a certificate of death, a doctor has to analyse a particular instance of the genus 'death,' and to place it under one of the species into which that genus is divisible. If that species be such an one as disease or poison, the analysis has to be carried further to determine the particular disease or poison involved. That this is not an easy piece of induction is shown by the fact that several doctors examining the same case may come to different conclusions; or, more accurately, form different hypotheses.

What does such an investigation imply? Surely that the total event 'death' can be analysed into co-operative factors working in certain bodily conditions, and that a constant relation holds between certain definite agents and certain elements in that effect. Some of these have already been established. When all are certain, medical diagnosis of the cause of death will be certain. It is when the elements cannot be clearly distinguished, so that there is room for doubt

as to the nature of some of them, that difference of opinion is possible.

Supposing it rightly and thoroughly done, the particular case is no longer simply 'death.' It is a definite form of death which is in constant relation with an equally definite cause. Death from an over-dose of strychnine, for example, differs from death from an over-dose of laudanum; each differs from death from suffocation. All these can be distinguished from death by drowning, this again from death from a cut throat, from a bullet in the brain, and so on. Each cause of death has for its effect, not death pure and simple, but a specific form of death.

(iii) Reciprocal Relation of Cause and Effect.—This is a typical example. The problem of causation is not solved in any one case till the whole event has been so analysed that a constant relation is reached, in which each term implies the other. It is constant, because, given the cause, the effect inevitably follows; given the effect, it is always due to that cause. That is expressed by the principle of causality: *Same cause, same effect*; and reciprocally, *Same effect, same cause.*

So long as the analysis is not made, or is imperfectly made, it may seem that one and the same effect may be due now to one cause, now to another; that there may be 'alternative causes,' or, as it is sometimes expressed, 'a plurality of causes.' As the analysis proceeds, first one, then another, of the alternatives is shown to be out of the question. When it is complete, only one possible cause remains.

For practical purposes we are most interested in knowing what effect a certain action will produce, and our purpose is often served by securing a general effect. In earlier days a ruler often wished to 'remove' an inconvenient subject. It was a matter of indifference to him how the removal was effected, so long as it was done. A railway engineer may need to blast a tunnel through a hill. The blasting agent

may be dynamite, or gunpowder, or some other explosive. That is of minor importance. The general effect of making a passage is the same, whatever agent is used. But that effect is susceptible of analysis, and each explosive causes a different kind of effect.

So we see why the idea of a possible choice of causes marks a preliminary stage in the establishment of a causal relation. We also see that the relation is not fully established till this possibility of alternative solutions of the problem is removed by deeper analysis.

In another sense it is more legitimate to speak of 'a plurality of causes' as producing a given effect. The effect may be due to a combination of causes acting together. Thus, the variation in the height of a barometer is due partly to the variation of atmospheric pressure, partly to the variation of the expansion of the column of mercury caused by heat. Calculation or experiment must decide how much of the joint effect is due to each of the co-operating causes. Similarly, the line of motion of a solid body may be due to the composition of several forces acting upon it in different directions.

(iv) Cause and Conditions.—An effect is secured only in appropriate conditions. A small boy sees a miner apply a light to a train of powder leading to a charge inserted in a rock; the train catches fire; an explosion, with fall of rock, follows. He imitates this with a train of sand: he applies the lighted match; the sand remains unaffected.

If the firing of the train be called the cause of the explosion, the nature of the powder is assumed as an indispensable operative condition. By itself it is quiescent; by itself the use of the lighted match is ineffective. It is the bringing the latter into close spatial relation with the former that secures the explosion.

The explosion tears the rock, the large masses fall. Again, the nature of the rock determines the kind and amount of the rending. That rending, by destroying the cohesion which

kept the rock whole, leaves free play to the attractive force of gravity, and the pieces fall.

We should be likely to say that the explosion caused the fall. We name the beginning and the end of an occurrence 'cause' and 'effect,' especially when our own agency starts the chain, and the end is a fulfilment of our desires. But we have not analysed the process. The more closely we do so, the more intermediate changes we can distinguish. Could we make the analysis complete we should see, not a series of separate happenings, but a constant and continuous transfer of energy.

Where we draw the line between cause and effect depends on the point of view. The miner probably places it at the application of the match to the fuse; the mining engineer at the ignition of the charge; the director of the blasting operations at the shattering of the cohesion of the rock. But, wherever it is placed, the relation is not known unless all that co-operates to provide just that effect is taken into account.

Sometimes it is all called the cause; at others, the state of things which by itself would remain unchanged is called the conditions, and the action which initiates the change, the cause. The latter agrees more closely with general popular usage; the former approaches more nearly the conception of the constant transformation of energy. But however the operative factors are designated, the investigation of a causal relation must take them all into account, and set forth the part played by each.

Nor is this analogous to sorting out the threads in a skein of wool, or even in a piece of wrought tapestry. The factors do not always combine mathematically, as in the composition of mechanical forces. Their union may change each, as in a chemical combination, in which the qualities of the compound cannot be deduced by adding together those of the elements. This gives rise to one of the great difficulties of inductive

investigation, especially when it starts with an effect, and seeks to determine its cause.

(v) Effect and Result.—An effect is a change produced by a cause. That change may endure long after the causal activity is over. The fallen rock remains on the ground indefinitely, unless it is removed in another chain of causal relations, having men and horses as its salient agents, and waggons among its operative conditions. Such a permanent effect is rightly called a 'result.' The fall of the buildings of Louvain was the effect of the German bombardment; the result was a state of ruin, which will remain unchanged until it is made a term in another causal series, either of further destruction or of restoration and rebuilding.

(vi) The Pure Case.—The investigation of a causal relation, then, is always a work of analysis. It seeks to eliminate all that is immaterial to the existence of the suspected relation, and to lay bare a *pure case* of cause and effect, into which no extraneous elements enter.

3. Suggestion of Causal Relations.

(i) Not subject to Rule.—The suggestion of a causal relation may be made in all sorts of ways. To a mind interested in a subject, full of pertinent knowledge, with power of imaginative construction, many things which other minds would pass unnoticed will suggest a hypothesis of causation. "The very existence of galvanism, or electricity of low tension, was unsuspected until Galvani accidentally touched the leg of a frog with pieces of metal. The decomposition of water by voltaic electricity also was accidentally discovered by Nicholson in 1801, and Davy speaks of this discovery as the foundation of all that had since been done in electro-chemical science."[1]

An observed regularity in the course of events naturally

[1] Jevons: *The Principles of Science*, p. 530.

raises the question as to whether they are causally related. It is true that no sequence is more regular than that of day and night, and that nobody takes either to be the cause of the other. Yet we all know that the sequence is due to a causal system, and is the necessary effect of the rotation of the earth.

However suggested, the mind at once seeks for an analogy, in the light of which it may work to test the suggestion. For example, the ancient Greeks knew that sound is due to vibrations acting on the drum of the ear, but the kind of vibration was first suggested by Newton on the analogy of wave-motion in water. The analogy of sound suggested the undulatory theory of light, even though to apply it the transmitting medium had to be assumed.

(ii) Method of Residues.—Hypotheses in science are often suggested by what has been called the *Inductive Method of Residues*.

Causal relations are not found separate in the world. In seeking them we must begin—as in every other exercise of thought—by selecting a small piece of the real world for examination.

Now, if we knew exactly all that was concerned in the case, if, moreover, we could know that all the causal elements but one accounted for all but a small part of the effect, we could immediately infer, without much doubt, that the remaining causal element was causally connected with the residual part of the effect.

If Jones and Brown sat next each other in an examination in arithmetic, and no one else was near; if Jones gave up his paper when Brown had worked five out of the six sums set; if Brown showed the same right solutions as Jones to the first five sums, and in the sixth was egregiously wrong; this residual phenomenon would be likely to be attributed by the master to Brown's own ingenuity. For only two causes could be supposed operative—copying from Jones, and native

genius. The former accounts for the five correct solutions, then that cause is removed and the corresponding effect ceases. Another kind of effect occurs, and only one possible cause remains to explain it. Brown stands condemned.

Such a simple instance as this is of no scientific interest, and establishes no generalised causal law. It explains the particular case, and in doing so brings Brown's conduct under general relations. There it ends.

In the search for constant relations of physical causation we have no such easily separable causes. So every residual phenomenon propounds a problem.

Perhaps the most striking instance in the history of science is the suggestion of the existence of the planet Neptune. "Adams and Le Verrier, working . . . independently . . . had observed certain perturbations in the planet Uranus. It did not keep in its proper orbit as determined by their mathematical calculations based upon the presence of the known stellar bodies."[1]

Here was the problem. Certain movements of Uranus were not accounted for by known causes. Analogy suggested the probable explanation of an outer planet as yet unknown to astronomers.

Then followed deduction of consequences from this hypothesis. "Adams and Le Verrier proceeded to calculate the exact position of such a disturbing body as determined by the nature and magnitude of the perturbations of Uranus."[2]

The next step was verification by comparison with new facts. The portion of the sky thus indicated was searched with the telescope, and the new planet was revealed to observation.

The discovery in the atmosphere of such inert gases as argon resulted from an investigation started by the detection of the unexplained residual fact that what had been taken as

[1] Hibben: *Inductive Logic*, p. 147. [2] *Ibid.*

the pure nitrogen of the atmosphere was about $\frac{1}{2}$ per cent. heavier than nitrogen obtained from various chemical compounds, which was always of uniform density.

As knowledge becomes more exhaustive, the part played by small unexplained facts in suggesting lines of inquiry grows increasingly important. "It is here, perhaps, that in the present state of science we may most reasonably look for extensions of our knowledge; at all events, we are warranted by the recent history of natural philosophy in so doing. Thus, . . . the peculiar smell observed in a room in which an electrical machine is kept in action was long ago observed, but called the 'smell of electricity,' and thus left unexplained. The sagacity of Schönbein led to the discovery that this is due to the formation of ozone, a most extraordinary body, of enormous chemical energies; whose nature is still uncertain, though the attention of chemists has for years been directed to it."[1]

4. Methods of Investigating Causal Relations.

(i) Aim.—The problem being suggested and a hypothesis formed through analogy with some known causal relation which presents what seem to be material points of likeness, the inductive inquiry must proceed to attempt to prove it. When the relation suggested is between facts directly open to observation, certain direct methods are available. The aim in all of them is to isolate the causal relation, if possible physically; in any case, in thought. They thus seek to shut out from consideration all elements in the complex event that are irrelevant. The assumption is that any element which can be absent when the effect is found, or present when the effect is absent, is not part of its cause.

(ii) Agreement.—"So long ago as the time of the Greeks it was already known that amber, when rubbed, will attract

[1] Thomson and Tait: *Elements of Natural Philosophy*, vol. i., pp. 113-114.

or draw towards it bits of straw and other light bodies, and it is from the Greek word *electron* = amber, that our word electricity is taken.

Until the sixteenth century, however, no one had made any careful experiments upon this curious fact, and it was Dr. Gilbert, a physician of Colchester, who first discovered that other bodies besides amber, will, when rubbed, attract straws, thin shavings of metals, and other substances. . . . Gilbert showed that amber, jet, diamond, crystal, sulphur, sealing-wax, alum, and many other substances, have this power of attraction when they are rubbed." [1]

The substances were chosen out of a wide range. The rubbing was constant; the effect was constant. All else was variable. So the probability that the rubbing was causally connected with the appearance of the power of attraction became higher with each new experiment. And the increase was greater than the mere addition of the separate pieces of evidence, for their variety gave them cumulative force.

This mode of gathering and estimating evidence is known as the method of *Agreement*. Its principle is simply that when two events—such as the rubbing and the attraction—accompany each other in different surroundings, they are probably causally connected, and that probability gathers strength with each new variation of circumstances.

When it was noticed that malaria was common near marshy lands, a causal connexion was suggested. The influence of damp was the first obvious hypothesis. But dampness is not always attended by the disease. So that hypothesis was rejected because the agreement is not constant. When further investigation showed the invariable presence of a certain mosquito in places subject to malaria the hypothesis that the insect is an active agent was sug-

[1] Miss A. B. Buckley: *A Short History of Natural Science*, pp. 74-75.

gested. But proof was reached only when it was shown that the bite of the mosquito injects a parasite into the blood, to which the disease owes its origin.

We constantly use the method in the ordinary affairs of life. If ornaments are broken, furniture scratched, curtains torn, wherever and whenever a certain servant cleans a room, we attribute the mischief to her careless outputs of energy.

The reader can easily find numbers of examples of the more or less precise application of the method. That it leads at times to wrong conclusions shows that by itself it can only bring additional *evidence* in support of a hypothesis, and so strengthen the probability of its truth. A hypothesis can be *proved* only by inferring consequences of a different kind, and bringing them to the test. Neglect of this leads to much fallacy in assuming to be certainly true what is only shown to have some probability in its favour.

The full process of first gathering evidence by the method of agreement, and then proceeding to verification is seen in Count Rumford's investigations into the nature of heat.

"Most scientific men looked upon heat as a fluid, which they called *caloric*, until, in the year 1798, Count Rumford first showed by experiment that it is probably a kind of motion."[1] The hypothesis was suggested to him by noticing, when boring cannon, the great heat produced by the grinding of the borer against the gun. "This led him to consider how it could possibly happen, if heat were a fluid, that the mere rubbing of two metals together should produce it; and he tried many experiments to find out whether the gun, the chips, or the borer had lost anything in consequence of having given out heat. But he could not discover that they were changed in any way; and moreover, he found that by going on boring he could make them give out heat as long as he liked, whereas if he had been drawing a fluid out of

[1] Miss A. B. Buckley: *Op. cit.*, p. 342.

the metals it seemed to him that it ought to come to an end sooner or later. Then he considered whether the heat could come out of the air, and to avoid this he repeated the experiment under water, but still the metals grew hot, and even made the water warm, so it was clear they had not drawn any heat from that fluid."[1]

When we consider this set of experiments we see the one constant connexion of friction and heat appearing under a great variety of circumstances, so planned that they tested other suggested explanations. Each was a crucial experiment—*experimentum crucis* as Bacon called it—for each negatived a rival supposition. The whole set was fatal to the hypothesis that heat is a fluid.

The method of agreement, then, had shown that the explanation suggested by the development of heat during the boring of the cannon was very probably true.

Now for the verification. Rumford inferred that if the hypothesis were true "then by great friction he ought to be able to produce any amount of heat, and to prove this he tried the following experiment:

He took a large piece of solid brass the shape of a cannon, and partly scooped out at one end. Into this he fitted a blunt steel borer, which pressed down upon the brass with a weight of ten thousand pounds. Then he plunged the whole into a box holding about a gallon of water, into which he put a thermometer, and fastening two horses by proper machinery to the brass cylinder he made them turn it round and round thirty-two times in a minute, so that the borer worked its way vigorously into the brass. Now notice what happened: When he began the water was at 60° F., but it soon grew warm with the heat caused by the friction of the borer against the brass. In one hour it had risen 47 degrees up to 107° Fahr.; in two hours it was at 178°, and at the end

[1] *Ibid.*, p. 343.

of two hours and a half it actually boiled."[1] Davy gave a further verification when he caused two pieces of ice to melt by rubbing them together.

(iii) Agreement in Presence and Absence.—If every form in a school is disorderly whenever it is taken by a certain master, and orderly with every other teacher, the head master finds the explanation in the man rather than in the boys. The agreement in the presence of the conjoined events in varied circumstances is supported by agreement in the absence of both together in circumstances in all else as like as possible to the other set. Not only are there the positive instances of disorder conjoined with the attempted exercise of authority by one master, but the confirmatory negative ones that with every other master the disorder is absent.

The principle, then, is that if the exclusion of the supposed cause is found to be regularly attended by the exclusion of the effect, the causal relation has probably been discovered. So the method is often called one of *Exclusions*.

It gives a greater strength to the probability of a hypothesis than does the method of agreement alone. That always leaves room for a number of hypotheses, attributing the effect to one cause in one case, to another in another; but this keeps definitely to the evidence bearing on one hypothesis alone. It thus makes the suggested relation stand out in thought as an approximately pure case.

The hypothesis that the colours of animals have a protective value makes much use of this method. We give an example—

"It is well known that the 'praying Mantis,' Mantis religiosa, occurs in Italy in a green and a brown form. The former is usually to be found on green grass, the latter on herbage browned by the sun. Mr. Cesnola tied down among

[1] *Ibid.*, pp. 343-344.

green herbage twenty green Mantis, and among withered grass a similar number of brown individuals. After seventeen days they were all alive. He also tethered twenty-five green Mantis among brown herbage, and they were all dead after eleven days. The converse experiment was also made, forty-five brown Mantis being exposed on green grass, and of these only ten survived at the end of seventeen days. Most of the Mantis were killed by birds; five of the green ones were killed by ants. Here, then, is a proof, quite conclusive though the numbers are small, of the selective value of the protective coloration of both races of Mantis. If green Mantis and brown Mantis be exposed on green grass, the green ones will survive rather than the brown, the death-rate will be selective. Such a simple experiment gives more solid support to the view that protective coloration is due to natural selection than any accumulation of probabilities."

To establish the relation as general, further verification is needed.

"It is of enormous importance that cases similar to the above should be accumulated, so that stability may be given to the theory of natural selection by actual evidence that the survivors survive and the eliminated are eliminated because of some differentiating peculiarity or peculiarities. Hence a few more examples may be given.

Poulton fastened 600 pupae of tortoise-shell butterfly to nettles, tree-trunks, fences, walls, and so on. At Oxford, the mortality was 93 per cent. and the only pupae that survived were on nettles, where they were least conspicuous. In the Isle of Wight, the elimination was 92 per cent. on fences, as against 57 per cent. among nettles. Here, again, there was definite evidence of discriminate elimination. . . .

Professor Davenport, of the Carnegie Institution for Experimental Evolution, had 300 chickens in a field, 80 per cent. white or black and conspicuous, 20 per cent. spotted

and inconspicuous. In a short time twenty-four were killed by crows, but only one of the killed was spotted." [1]

An application of the method to practical affairs is seen in the following:—

"When we find an epidemic of typhus in a town, and one part of the town contains all those attacked and another part none; if all the infected houses draw their water from one reservoir *A*, while no case is found among those supplied from the second reservoir *B*; and if, in addition, the same facts should occur in a second town at a considerable distance, then we should have an indication that the conditions of the illness lay in the first reservoir of water. For all the cases of illness, however different their circumstances may be otherwise, would then agree in the fact that they have for their antecedent the use of water from source *A*; and where under similar circumstances of locality, climate, etc., the result is absent, the antecedent is also absent."

(iv) DIFFERENCE.—Pursuing the inquiry: "If it also happened that when the first reservoir of water was cut off no more cases of illness occurred, then the first result would be confirmed in another way." The selection we had made in thought is made in fact. The supposed source of the disease is stopped. If no more cases of typhus occur a causal relation is established between the water from *A* and the outbreak of the fever.

Can this be generalised as a constant relation? The same methods are applicable. "If it should then be found that the waters from the two sources differ in the fact that the first is defiled by refuse, then we have singled out a definite circumstance connected with that condition; and if in the second town, the other circumstances being quite different, the partial epidemic should again attack a district supplied from a source similarly defiled, then the proof that such

[1] Geddes and Thomson: *Evolution*, pp. 163-164.

defiled water is, or contains, the cause of the illness would be sufficiently complete. The fact that all do not fall ill who use the water can be explained by the general observation that certain individuals are not liable to such infections."

What, exactly, has been established? Only that "the impure water is, or contains, the cause of the typhus; we have not proved the universal proposition that wherever typhus appears impure water has been drunk."[1]

The inference has made use of what is known as the method of *Difference*, because its principle is that the suspected cause is absent in one case, and present in another, in circumstances otherwise identical.

If one of two adjacent flower-beds is treated with a certain kind of manure, and the same kind of plants sown in both, any difference in the richness of the flowers in the next season may be attributed to the influence of the manure.

If a tile falls on my head, I am sure the blow is the cause of the consequent headache.

If a man in good health eats tinned salmon and soon after is seized with violent internal pains, he has no doubt to what his sufferings are due.

If, on turning the switch for an electric light no light follows, there is defect in either wire or lamp. How am I to decide? I change the lamp, and if the light follows I infer that the defect was in the lamp.

Generally, if we add an active element we look for a change in the outcome; if we remove such an element we expect a different kind of change.

Messrs. Geddes and Thomson give an interesting example: "Mr. J. T. Cunningham put very young flounders in an aquarium lighted from below, and observed that as they underwent their peculiar metamorphosis the pigment first disappeared as usual from the down-turned side, and then

[1] Quotations from Sigwart: *Logic*, vol. ii., p. 420.

(in 11 cases out of 13) reappeared under the unusual stimulus of light from below. This shows that the normal absence of pigment on the down-turned side of a flat-fish is due to the absence of the light-stimulus in each individual case."[1]

The following experiment of Graber to prove that insects can hear was an application of the method of difference: "He placed some water-boatmen in a deep jar full of water, at the bottom of which was a layer of mud. He dropped a stone on the mud, but the beetles, which were reposing quietly on some weeds, took no notice. He then put a piece of glass on the mud, and dropped a stone on to it, thus making a noise, though the disturbance of the water was the same as when the stone was dropped on the mud. The water-boatmen, however, then at once took flight."[2]

Nevertheless, that a change follows on an action does not in itself prove connexion. We must allow for 'the long arm of coincidence.' "During the retreat of the Ten Thousand a cutting north wind blew in the faces of the soldiers; sacrifices were offered to Boreas, and the severity of the wind immediately ceased, which seemed a proof of the god's causation."[3] The desired change followed the adoption of the means traditionally held to be effective. The Greeks believed that the causal relation was established. Why do we reject it? Not because we deny the facts, but because we bring changes of wind under a wide system of meteorological laws, among which the influence of Boreas has no place. To the Greeks those laws were unknown, and the constant interference of the gods in mundane affairs was axiomatic.

(v) Concomitant Variations.—If my room is lighted by gas I must turn the tap to get a light. Again, I must turn it to put out the light. So, by the method of difference, it would be suggested, apart from any knowledge of how the

[1] *Op. cit.*, pp. 61-62. [2] Hibben: *Op. cit.*, p. 115.
[3] *Ibid.*, p. 323.

light is produced, that it is connected with the position of the tap. An electric switch illustrates the same point, and in that case many people do not know why the turning of the switch affects the light.

In the case of the gas we have a further relation which is absent with the electric switch. If we slowly turn the tap, the light gradually increases to a maximum; a reverse turning diminishes the light, till it goes out. So we can infer a connexion between amount of light and amount of turning, even before we know, from study of the mechanism of a tap, how the variation is produced.

This is a homely example of a most useful kind of evidence of a causal relation. It is called the method of *Concomitant Variations*, and its principle is that concurrent variation of two changes is probably due to a causal connexion between them.

Often a suspected cause cannot be removed, nor can we find cases of its absence. This is so, for example, with heat and gravitation. But we can vary the amount of their influence, and when the supposed effect varies regularly with this variation, we have evidence in support of a hypothesis of causal relation.

The space traversed in a second by a falling body varies with the height above the sea-level. At the top of a mountain it falls rather less than the sixteen feet through which it passes at the foot. Newton took the moon as an example of a falling body 240,000 miles above the surface of the earth, and by rigid calculation, and observation of the moon's orbit, found that "at the distance of the moon a body, so far from falling a distance of sixteen feet in a second of time, would commence its long journey so slowly that a *minute*, instead of a *second*, would have elapsed before the distance of sixteen feet had been accomplished." [1]

[1] Ball: *Story of the Heavens*, p. 100.

The change of the states of material bodies from solid, through liquid, to gaseous, as increasing amount of heat is applied, is evidence that the change is an effect of the increase of heat.

Concomitant variation must not be assumed to go on indefinitely. The amount of work a man turns out varies with the time he spends at it, but only within limits set by his power to resist fatigue. Because a healthy man can do twice as much work in four hours as in two, it must not be assumed that if he worked sixteen hours he would do four times as much as in four. It is frequently found that diminution of very long hours of manual labour increases the output, though, of course, this has also its limits.

Moreover, breaks in continuity may occur in physical relations. Water contracts with increasing cold till it reaches 4° Centigrade. Between 4° C. and 0° C. there is a slight expansion. At 0° a sudden expansion takes place, and ice is formed which, bulk for bulk, weighs less than water.

Nor need there be any equality in the amount of variation in cause and effect. "It has been found by experiment that a current moving at the rate of three inches per second will take up and carry along fine clay; moving six inches per second, will carry fine sand; eight inches per second, coarse sand the size of linseed; twelve inches, gravel; twenty-four inches, pebbles; three feet, angular stones of the size of a hen's egg. It will readily be seen that the carrying power increases much more rapidly than the velocity. For instance, a current of twelve inches per second carries gravel, while a current of three feet per second, only three times greater velocity, carries stones many hundred times as large as grains of gravel."[1]

It is evident, then, that though concomitant variation suggests a causal relation it is not sufficient to show its

[1] Hibben: *Op. cit.*, p. 329.

character. Its most fruitful application is to cases in which the intensity of the causal force is under control, and the variation in the effect can be measured. "To determine correctly the efficiency of windmills, when the natural winds were constantly varying in force, would be exceedingly difficult. Smeaton, therefore, in his experiments on the subject, created a uniform wind of the required force by moving his models against the air on the extremity of a revolving arm. The velocity of the wind could thus be rendered greater or less, it could be maintained uniform for any length of time, and its amount could be exactly ascertained." [1]

(vi) Employment of Methods.—For the sake of clearness, the methods have been set forth separately, but nobody would think of restricting himself to any one of them in any inquiry. They are typical ways in which the inductive worker tries to get a 'pure case'—to see the relation he is examining unencumbered by extraneous matter. Now one, now another, is pressed into the service. This is seen in the investigation into an outbreak of typhus, and, indeed, may be expected in any scientific inquiry.

Pasteur's investigations into the problem of the spontaneous production of life show both the use of all the methods and the great difficulty of excluding the possibility of error.

Briefly, he assumed as a hypothesis to be tested that the asserted spontaneous generation of living organisms in putrefying or fermenting matter was due to the importation of germs from without. Such germs are present in great numbers in the air. So, by exposing fermentable liquids to the air, he showed by the method of agreement that when the germs could fall on the liquids the living organisms appeared. By placing the same liquids in closed vessels and thus excluding the air, he obtained the corresponding negative

[1] Jevons: *Principles of Science*, p. 441.

instances, and showed that absence of air meant absence of the appearance of life. To the positive method of agreement he added those of agreement in absence and of difference. The difficulty of excluding all possibility of the importation of germs made these experiments very delicate, and Pasteur had to meet experiments which seemed to support the hypothesis of spontaneous generation, by showing that in none of them had the communication of germs from the air been made impossible.

By exposing the liquids to still air in caverns where the germs would long ago have fallen to the ground, and to air at various heights on mountains where the number of germs would decrease with the height, he showed that the evidence of concomitant variation supported his hypothesis that the apparent spontaneous appearance of life is always due to the propagation of an already living organism.

(vii) FORMAL ANALYSIS OF METHODS.—It is evident that all the methods seek to eliminate everything that is not pertinent. Sometimes this can be done in fact; always by manipulation of the facts the aim is to do it in thought.

The formal principle is that if a definite set of conditions result in a certain event, and if another set of conditions, partly like and partly unlike the former, lead to a different result, a causal relation probably exists between the elements found in both sequences, and not between the others. Symbolically—

If AB is followed by xy,
and AC ,, ,, xz,

then a causal relation probably exists between A and x; but not between A and either y or z, nor between B and either x or z, nor between C and either x or y. The possibility of such a relation between B and y, or C and z is neither supported nor disproved.

Applying this general analysis to the separate methods, we may symbolise them thus—

(*a*) *Residues*—

ABC is followed by xyz,

But $B\ldots y$ and $C\ldots z$ are known to be causal relations; therefore, the cause of x must be found in A.

If A is not known, it must be assumed, and steps taken to verify the assumption.

(*b*) *Agreement*—

$$\left.\begin{array}{llll} ABC \text{ is followed by} & & & xyz \\ ADE & ,, & ,, & xmn \\ AFG & ,, & ,, & xsv \end{array}\right\} \therefore A\ldots\ldots\ldots x \text{ is probably a causal relation.}$$

(*c*) *Agreement in Presence and Absence*—

$$\left.\begin{array}{lllll} & ABC \text{ is followed by} & & & xyz \\ & ADE & ,, & ,, & xmn \\ \text{But} & BDE & ,, & ,, & ymn \\ \text{and} & CDE & ,, & ,, & zmn \end{array}\right\} \therefore A\ldots\ldots\ldots x \text{ is probably a causal relation.}$$

(*d*) *Difference*—

$$\left.\begin{array}{lllll} & ABC \text{ is followed by} & & & xyz \\ \text{But} & BC & ,, & ,, & yz \end{array}\right\} \therefore A\ldots\ldots\ldots x \text{ is probably a causal relation.}$$

(*e*) *Concomitant Variations*—

$$\left.\begin{array}{llll} ABC \text{ is followed by} & & & xyz \\ A_1BC & ,, & ,, & x_1yz \\ A_2BC & ,, & ,, & x_2yz \end{array}\right\} \therefore A\ldots\ldots\ldots x \text{ is probably a causal relation.}$$

Such symbolic statements must be interpreted very guardedly. They suggest that the elements considered are easily distinguishable from each other, and that one can be present or absent without affecting the other parts of the complex whole in which it occurs. Neither suggestion is generally true. Consequently, they serve no further purpose than to lay bare the aim of eliminating elements not material to the problem.

CHAPTER XII.

VERIFICATION OF HYPOTHESES.

1. Nature of Verification.

(i) Outcome of Use of Direct Methods.—The methods of examining facts outlined in the last chapter are ways of testing suppositions by evidence. The evidence is not always of the same force. It is weakest when we have only the positive instances of the method of agreement; much strengthened when we can support these by corresponding negative instances; strengthened yet more when in a typical case we can exclude or introduce the suspected cause at will, and find that, in doing so, the supposed effect is removed in the one case, and secured in the other; made still more convincing when variation in the assumed effect attends changes in the assumed cause.

Such evidence may amount to a 'practical proof' that a relation exists. The relation has been separated in thought, and, as far as possible, in fact, from everything that seems able to interfere with it. We feel sure that it is true that a certain condition or change is the effect of the operation of a certain force.

But we cannot yet explain why it should be so. What is the bond which unites the cause and the effect, so that the occurrence of the one necessitates that of the other?

(ii) Need for further Analysis.—To answer this question we must push analysis further. We must find what qualities there are in the cause, and what in the effect, and

try to trace a relation of dependence between them, to see in their connexion an instance of a wide and well-established law. We must pass from a clear knowledge of the relation as a whole to a distinct knowledge of its nature.

Only when this step has been taken do we understand the causation; just as we understand the nature of things only when to the clear knowledge of them set forth in classification we add the distinct knowledge expressed in definition.

For example, observation in many lands in the present, and records of the past, agree in showing that wholesale and indiscriminate alms-giving and a degraded condition of the poor are always found together. It is a strong case of the method of agreement, and the evidence of the facts makes the existence of a causal relation very probable. But why should it be so? Why should acts of charity have the opposite effect to that intended? Nothing but analysis of the relation of human nature to its surroundings can give the answer. This shows that exertion and industry are stimulated by need, that thrift is an outcome of self-reliance, that independence of character prompts to the acquirement of the means of comfort without reliance upon extraneous help. The special case is explained by being brought under a number of universal laws of human nature. Man being what he is, the causal relation is seen to be inevitable.

Count Rumford's experiments on the cause of heat led on to such an explanatory verification. If friction be the cause of heat, then it must follow the general law that amount of heat varies with its cause.[1]

(iii) Limitation of Use of Methods.—The methods can be applied only when the causal relation is directly apparent in facts of experience. Were we confined to them, we could never get behind description to explanation. For explanation involves a reference to those wide relations which

[1] Cf. pp. 209-210.

are known to us only in their results; as in the analysis of human nature which gives the key to the dependence of pauperisation upon injudicious almsgiving.

How do we know gravitation? Only in its effects, and the progressive understanding of this all-embracing relation was a continuous passing from its smaller and more open manifestations, such as the fall of a stone or an apple to the ground, through wider and less apparent instances, such as the motions of the planets, to the widest conception of an attraction which holds according to the same law between all parts of matter. In that we have got as far away from what is discernible by the senses as we have in the analysis of the effects on human nature of indiscriminate doles.

All the great theories of science are of this kind. In themselves we cannot examine them; we know them by their effects. Such knowledge is not given by direct observation of facts; but, nevertheless, it is related to such observation. The facts call for explanation. The mind replies by imagining one.

There is a temptation to stop there; to take the hypothesis for proven truth. This is to act like Don Quixote, who, having made a cardboard helmet, impressive in appearance, refused to submit it to the ordeal of blows. Like the helmet, our unverified hypotheses are likely to fail us in the hour of need.

The accepted undulatory theory of light is bound up with the assumption of an ether, of which no direct experience is possible. But its qualities are inferred from facts open to observation. The history of the earth in the far past can be learnt only by interpreting the structure and arrangement of its crust in the present. In the investigation this involves, are found signs, in definite surroundings, which can be interpreted only as witnessing to the lives and doings of men in far more remote ages than previous thought had conceived possible.

Verification of all such theories must be sought in the agreement of their necessary logical consequences with facts open to observation. The process of inference between widest theory and simple fact may be a long one, only to be taken in many steps, and often occupying many successive years. But to stop short of the facts is to leave the theory suspended in the air, like Mahomet's coffin, without support.

We take a simple case for illustration. At the beginning of the seventeenth century, Fabricius Aquapendente discovered the valves in the veins, and supposed their use to be simply to prevent the blood escaping too quickly into the branches of the veins. Harvey, who was studying under him at Padua, was not satisfied with this suggested explanation. So he first experimented on a living animal to get an exact knowledge of the facts. By tying first an artery, then a vein, he proved that the blood in the former flows from the heart, and that in the latter towards it. This suggested the hypothesis of the circulation of the blood, from the heart by the arteries, through all parts of the body, back again to the heart by the veins. The passage is not open to observation in all its course, and Harvey worked at the problem for nineteen years before he was confident that what is open to direct observation confirms the general hypothesis, and is inconsistent with any other.

The hypothesis that acquired characteristics are not transmitted from one generation to another can be tested by observation and experiment only indirectly, by determining whether facts agree or disagree with consequences which can be inferred from it. "For example, many butterflies have two generations in the year, one of which lives through its whole life-history in the summer and the other passes the winter as a pupa (chrysalis). In some cases the two generations are strikingly different, and it has been shown that by freezing the pupæ of the summer brood at the right stage, specimens like the spring brood can be obtained. The

difference between the two generations is thus due to the action of cold on the pupa. But the two forms regularly alternate in nature and the effects of cold are not inherited. In plants, some species produce quite different leaves according to whether they are grown in water or in dry soil, but the conditions act on the individual, and do not affect its progeny. In such a case, what is inherited is the faculty of making a certain definite response to definite conditions, and this faculty is present whether the conditions operate or not. In man such diseases as tuberculosis are commonly called hereditary; this however does not mean that the child has the disease because his parent had it, but that the parent had a constitution liable to that disease, and the child inherits a similar constitutional liability. If the parent had never been exposed to infection the child would still inherit the liability, for what is transmitted is not the disease or its effects, but the faculty of acquiring it if exposed. It will be found that most cases which at first sight seem to support the theory of the inheritance of acquired characters are equally explicable in the view that . . . what is inherited is not the character acquired, but the innate power of acquiring it."[1]

(iv) Work of Thought in gaining Knowledge.—Inference of consequences from hypotheses is, then, an indispensable and most important step in reaching knowledge. The crude belief that man has only to look and he will learn is seen to be false as soon as we examine fairly how knowledge has grown, and is growing. It is true that 'facts are the beginning and the end of knowledge.' But between that beginning and that end, two intermediate steps must be taken. If not, the passage is not made; beginning and end are identical; we are ever at the starting-point. Those two steps are taken not by any of the senses, but by the power of thought.

[1] L. Doncaster: *Heredity*, pp. 25-26.

The first of them is the work of the imagination, generally guided by the detection by thought of pertinent analogies. That cannot be reduced to rule. We know it as a matter of cultivated native endowment, in which people differ enormously. With it we have already dealt.

The second is the inferring of the consequences of the suggestion offered by imagination. Only when that is done are we ready to make appeal to fact as the judge to pronounce sentence. That sentence is decisive. If any one fact is really inconsistent with the suggested explanation, that explanation must be given up. But if there is much positive evidence in its favour, we must make sure that the judge is both a real and an impartial one; that the inconsistent fact is true, and that it is truly interpreted. The final appeal to fact, however, is worthless, unless the step of inference has been well and truly taken.

2. Nature of Inference involved in Verification.

(i) MATTER AND FORM.—The difficulty of inference is always in the matter about which we reason. The process which forms the general framework is, in itself, always simple. So it is easy to understand it when it is expressed in symbols, and thus stripped of the difficulty due to the complexity of facts. The investigation of this framework is the work of logic. That structure shows the general principles which are embodied in the inference when it is valid.

To understand this will not enable us to reason well in any particular subject, because every subject fills out the framework with its own material. To do the filling properly we must know much about that material.

On the other hand, to be quite clear as to the nature of the framework is no small gain. Without this, though we may know the material well, we may so pile it on the framework that we break it down.

All this is to say that to think well we must both rightly

choose, and rightly use, our material. The former is a question of knowledge pertinent to the problem in hand; the latter one of clear apprehension of right modes of thinking.

(ii) Structure of the Inference.—The structure of the inference by which we verify hypotheses is determined by the nature of the problem. This is to bring together supposition and fact, that the latter may test the former. The appropriate setting of a supposition is the hypothetical proposition; that of a fact, the categorical proposition. Hence, our framework will bring these together, and the result will be to state as a fact that the hypothesis is true or false—another categorical proposition. Such a structure is called a *Mixed Hypothetical Syllogism.* It is an inference to the truth or falsity of one element in a hypothetical proposition from the affirmation or denial of the other element.

(iii) Forms of the Inference.—We take, then, a hypothetical proposition of the general form *If* **A** *is true,* **C** *is true,* where **A** is a statement of the assumed relation, and **C** one of the consequences which would follow from it. Both **A** and **C**, by themselves, are categorical propositions: **C** is a statement of fact open to observation; **A** may, or may not, be such a statement. The proposition as a whole asserts neither of them; it asserts such a relation to hold between **A** and **C**, that if **A** be secured **C** is also secured.

Does it follow that if **C** be found to be true in fact, the truth of **A** can be inferred?

Let us examine an instance: A man has been murdered, and much circumstantial evidence points to another man as the murderer. He is known to have been on bad terms with the victim; was heard to threaten him a day or two earlier; was seen in his company shortly before the crime was committed; would profit by his death. The case looks black against him. But at the trial he produces unimpeachable evidence that at the time the murder was committed he was at another place. His acquittal follows as a matter of course.

Let us set out the inference made by the jury—

If A is guilty of the murder of B, he was present at the place C where the crime was committed;

but A is proved not to have been at C at that time;

therefore A is not guilty of the murder of .

He is acquitted because he has proved a fact inconsistent with the hypothesis of his guilt.

Now suppose, on the other hand, that he was proved to have been present. Is that sufficient to establish his guilt? Not unless he is shown to have been the *only* person, besides the murdered man, then present. If several persons were there, as, for instance, if A was killed in a riot, all that can be asserted is that some one of them was guilty, not that a particular one was the culprit. The presence of A is only an additional piece of circumstantial evidence against him; it is not proof.

The fact that it is generally possible to form two or more hypotheses to account for a given event, shows that its occurrence is not a proof of the truth of any one of them. If we symbolise the event by **E**, and the suggested explanations by **X**, **Y**, **Z**, we have three hypotheses to be tested, of which the symbolic statements are—

If **X**, *then* **E**.
If **Y**, *then* **E**.
If **Z**, *then* **E**.

It is evident that proof that **E** is true does not establish the truth of either **X**, or **Y**, or **Z**.

Now, in attempting to verify any one of these hypotheses, **X**, we deduce from it other consequences than **E**, which we may symbolise by **C**. But this gives no assurance that **C** may not also be inferred as a consequence from **Y** or **Z**, just as **E** could be.

So we see what is meant when it is said that though a single assured fact is sufficient to overthrow a hypothesis

with which it is inconsistent, it is not sufficient to establish a hypothesis from which it is deduced as a consequence.

On the other hand, if in any way the hypothesis can be proved to be true, the consequences correctly deduced from it are thereby also established as true.

There are, then, two moods of mixed hypothetical syllogisms which yield an assured conclusion. *If we prove the antecedent* (**A**) *of the hypothetical proposition, we thereby prove the consequent* (**C**)*: if we disprove the consequent* (**C**), *we thereby disprove the antecedent* (**A**). In the first case, the inference is said to be *Constructive*, or in the *Modus Ponens*: in the second case, to be *Destructive*, or in the *Modus Tollens*. The general symbolic statements are—

Modus Ponens: *If* **A** *is true,* **C** *is true;*
And **A** *is true;*
Therefore, **C** *is true.*

Modus Tollens: *If* **A** *is true,* **C** *is true;*
But **C** *is false;*
Therefore, **A** *is false.*

But from *If* **A** *is true,* **C** *is true;*
And **C** *is true;*

or from *If* **A** *is true,* **C** *is true;*
But **A** *is false;*

we can draw no certain conclusion.

(iv) PROBABLE CONCLUSIONS.—Are these latter forms of reasoning, then, of no value? By no means. Formal logic, indeed, disregarded them; it acknowledged only such inferences as yield a certain conclusion. But this is a narrow restriction of the province of logic, which shuts out from its examination a vast amount of reasoning both in ordinary life and in science. We continually use such inferences, and in doing so we do well, provided that we recognise that the con-

clusions we draw are only 'probably' true, and that the degree of probability may vary from bare possibility to practical certainty. So long as we are on our guard against inferring more than is warranted by our premises, we are justified in reasoning to what is probable as well as to what is certain.

Inferences of these kinds do not yield *proof*, but they furnish *evidence*; and proof can often be approached only through the accumulation of evidence. We have seen that this may become so strong that it gives us a conviction of truth clear enough to guide our actions. At the same time it must be remembered that evidence is always liable to be overthrown by rebutting evidence; it never reaches the stage of strict proof.

In many cases the only verification attainable, it may be for many years, is the accumulation of such evidence as is symbolised by the inferences—

If **A** *is true,* **C** *is true;*
" " **D** *is true;*
" " **E** *is true;*
" " *etc.;*

and the successive showing that **C, D, E,** etc., all are true, while in no case is a consequence deduced from **A** that is disproved by appeal to fact. So the probability of the truth of **A** grows continually greater.

Our confidence is especially strong when the hypothesis not only explains the facts brought under it, but also fits into the general system of knowledge. "The ultimate criterion is the power of verifying conclusions, of testing truth by its capacity to explain phenomena, and by its conformity to the scientific truth already established beyond dispute."[1]

[1] Sir L. Stephen: *The English Utilitarians*, vol. iii., p. 503.

(v) Prediction of Events.—The consequences inferred are often predictions of events. When such a prediction is fulfilled, the evidence for the truth of the hypothesis is both strong and impressive. Examples are numerous.

Astronomical phenomena can be foretold with great accuracy, and the constant fulfilment of the predictions is an assurance of the truth of the theories from which they are made. For instance, in his account of the war between the Medes and the Lydians, Herodotus records that "in the sixth year, when they were carrying on the war nearly equally on both sides, they came to battle, and whilst they were contending for victory, the day was suddenly turned into night; which alteration Thales the Milesian had foretold to the Ionians, and named the year when it should happen."[1]

"There is a well-known story as to the happy use which Columbus made of the power of predicting eclipses in overawing the islanders of Jamaica who refused him necessary supplies of food for his fleet. He threatened to deprive them of the moon's light. 'His threat was treated at first with indifference, but when the eclipse actually commenced, the barbarians vied with each other in the production of the necessary supplies for the Spanish fleet.'"[2]

"Halley has the glory of having first detected a periodic comet, in the case of that which has since borne his name. . . . In 1705, Halley explained how the parabolic orbit of a planet may be determined from three observations; and, joining example to precept, himself calculated the positions and orbits of twenty-four comets. He found, as the reward of this industry, that the comets of 1607, and of 1531, had the same orbit as that of 1682. And here the intervals are also nearly the same, namely, about seventy-five years. Are

[1] *History*, bk. i., ch. 74. [2] Jevons: *Op. cit.*, p. 537.

the three comets then identical? In looking back into the history of such appearances, he found comets recorded in 1456, in 1380, and in 1305; the intervals are still the same, seventy-five years or seventy-six years. It was impossible now to doubt that they were the periods of a revolving body; that the comet was a planet; its orbit a long ellipse, not a parabola.

But if this were so, the comet must reappear in 1758 or 1759. Halley predicted that it would do so. . . . He predicted that the comet would reach its perihelion April 13, 1759, but claimed the license of a month for the inevitable inaccuracy of a calculation which, in addition to all other sources of error, was made in haste, that it might appear as a prediction. The comet justified his calculations and his caution together; for it arrived at its perihelion on the 13th of March."[1]

The discovery of the New World by Columbus was a verification of the consequences he had deduced from the hypothesis of the rotundity of the earth.

The history of every science furnishes examples of successful prediction. "Chemistry . . . has not been without prophetic triumphs. The existence of the metals potassium and sodium was foreseen by Lavoisier, and their elimination by Davy was one of the chief *experimenta crucis* which established Lavoisier's system. The existence of many other metals which eye had never seen was a natural inference, and theory has not been at fault. . . . The theoretical chemist by the classification of his specimens and the manipulation of his formulæ can plan out whole series of unknown oils, acids, and alcohols, just as a designer might draw out a multitude of patterns."[2]

For fifteen years a plague had raged among the silkworms in France. Pasteur was urged to investigate it, and after

[1] Whewell: *History of the Inductive Sciences*, vol. ii., pp. 182-183.

[2] Jevons: *Op. cit.*, p. 544.

doing so, he proved "that the plague-corpuscles might be incipient in the egg and escape detection. . . .

In 1866, he inspected at St. Hippolyte-du-Fort fourteen different parcels of eggs intended for incubation. Having examined a sufficient number of the moths which produced these eggs, he wrote out the prediction of what would occur in 1867, and placed the prophecy as a sealed letter in the hands of the mayor of St. Hippolyte.

In 1867, the cultivators communicated to the mayor their results. The letter of Pasteur was then opened and read, and it was found that in twelve out of fourteen cases there was absolute conformity between his prediction and the observed facts. Many of the groups had perished totally; the others had perished almost totally; and this was the prediction of Pasteur. In two out of the fourteen cases, instead of the prophesied destruction, half an average crop was obtained. . . .

Now, the parcels of eggs here referred to were considered healthy by their owners."[1]

(vi) Proof.—Confidence in a hypothesis grows with repeated confirmation of the consequences deduced from it. But strict proof is reached only when it is shown that no other explanation of the facts is admissible.

This, as we have seen, implies deeper analysis of the whole event. Death may be the effect of disease, poison, drowning, and of many other causes. The occurrence of a death does not prove that any particular one of its possible causes was operative. A post-mortem examination tries, by analysing a particular case of death, to attribute it to some one cause to the exclusion of all other conceivable causes. It aims at being able to say, not only that if a certain cause were present this effect would follow, but that this particular effect was certainly due to one definite cause.

[1] Tyndall: *Fragments of Science*, pp. 135-137.

Fully to establish a hypothesis, then, it is necessary to prove, not only the direct statement that certain consequences will follow from a particular hypothesis, but also the reciprocal proposition that those consequences can follow from no other supposition.

Symbolically, it has to be established, not only that *If* **A** *is true*, **C** *is true*, but also that *If* **C** *is true*, **A** *is true*.

How is this to be done? Here comes in the value of contraposition. The obverted contrapositive of *If* **C**, *then* **A** is *If* **not-A**, *then* **not-C**. So we have to seek cases in which the supposed cause is absent, and if we find the effect also invariably absent, we have proved that the causal relation between them is reciprocal: that neither can be found without the other. This is the object of the appeal to negative instances in the direct methods.

But we cannot always directly exclude the cause. How, then, can we be sure of its absence in any particular case? Well, we know that if it were present its effects must also be present. If the cause of a death be poison, the corpse will show certain characteristics at the post-mortem examination. If these are absent, the hypothesis of poison is negatived. So in succession every other hypothesis but one is disproved, because the peculiarities of the death which would result from them are not present.

But the characteristic marks of death, say from suffocation by the fumes of charcoal, are present, and these are compatible with no other hypothesis, because they differ from the characteristics of death from any other cause, and are themselves never found in such deaths. If that cause be assumed absent, the effect actually present would not be present. So, both the positive relation: *If* **A**, *then* **C**, and the negative relation: *If* **not-A**, *then* **not-C**, are shown to hold. That is, not only would suffocation cause death, but this particular form of death can be attributed to nothing else.

3. Dilemmas.

(i) Intellectual.—When two hypotheses are suggested, from each of which various consequences are deduced, all of which agree with observed facts, scientific theory is in a dilemma. There is no certainty which hypothesis should be taken as a guide. The uncertainty must continue till a case is found in which the consequences inferred from the one are incompatible with those deduced from the other. Then appeal to fact is decisive against one of them. If possible, the appeal should be made experimentally, so as to secure that no interfering factors are operative. That is the function of a crucial experiment.

Such intellectual dilemmas have been common in the history of science. We will give two examples—

(1) In 1674 Mayow published the theory that 'fire-air' is the chief agent in combustion. About the same time the 'phlogiston' theory was advanced by Becher and Stahl. "Stahl imagined that all bodies which would burn contained an invisible substance which he called '*Phlogiston*,' and that when a body was burnt it gave up its phlogiston into the air, and could only regain it by taking it out of the air or some other substance.

This theory . . . seemed to answer so well in a great many problems, that chemists believed in it for nearly a hundred years, and Mayow's true explanation was forgotten till the eighteenth century, when fresh experiments brought it again to the front."

About 1770 Lavoisier began to suspect the phlogiston theory, because "he found . . . that when metals are heated so that they turn into powder, the powder weighs more than the original metal did before it was heated. Moreover, he also found that the air which remained behind in the vessel in which the metal had been heated had lost exactly as much weight as the metal had gained. . . .

For eight years Lavoisier worked incessantly at this pro-

blem. . . . At last, in 1778, it occurred to him that Priestley had separated mercuric oxide into . . . mercury and a gas." So he first made mercuric oxide by heating mercury in a limited amount of air, and noted the proportion of the air that disappeared. He then separated the oxide back again into the metal and the gas. Thus, "the imaginary phlogiston, which had been supposed to load the air when anything was burnt in it, was proved never to have had any existence; for it was clear that just the opposite effect takes place."[1]

(2) Galvani "had prepared the hind legs of several frogs and hung them by copper hooks upon an iron balcony outside his house. As they hung there the wind swayed them to and fro, so that the ends of the legs touched the iron of the balcony; and every time they did so he noticed that the legs were convulsed just as they had been by the electrical machine and the lightning. But this time he could not see that any electricity had come near them from outside, so he supposed that there must be an electric fluid in the leg itself, which passed round every time the two ends of the legs were joined by the metal. These discoveries of Galvani soon became spoken of far and wide under the name of galvanism, and the supposed fluid was called the galvanic fluid. . . .

Not satisfied with merely reading about Galvani's experiments, Volta tried them himself, and he began to suspect that the electricity was not, as Galvani imagined, in the frog's leg, but was produced by the two metals, copper and iron, upon which the legs had been hung, and which were acted upon by the moisture in the flesh.

Then began a very famous controversy. Volta insisted that the electricity came from the *metals*, Galvani that it came from the *animal*. In each new experiment which Galvani brought forward to prove his point, Volta still showed that

[1] Quotations from Miss A. B. Buckley: *Op. cit.*, pp. 133, 235-237.

the electricity could be produced without the animal, until at last Galvani succeeded in finding a test which he thought must silence Volta for ever. He found that by laying bare a nerve of the leg of a frog, called the 'crural nerve,' and bringing the end of it to the outside of the muscles of the leg, he could produce the convulsions without any metal at all. But Volta was not so easily convinced; he still insisted that it was the different fluids and tissues being brought together which caused the electricity, and that there was not a current running through the animal. At this point, just when the truth would probably have been worked out, Galvani died (in 1798), leaving Volta in possession of the field; and for twenty-eight years no more was heard of animal electricity.

We know now that both the professors were right. Volta was right in saying that the convulsions of the frog's legs on the balcony were produced by the contact of the two metals in connexion with a fluid; while Galvani was right in saying that there is an electricity in animals which acts without any other help. In 1826 an Italian named Nobili repeated Galvani's experiment, and having then an instrument called a galvanometer, by which the passage of the faintest electric current can be detected, he proved that such a current does exist in the frog, and it has since been found to be common to all animals."[1]

In the first case the dilemma was resolved by the disproof of one of the alternative hypotheses; in the second, by finding that they were not incompatible with each other when stated with sufficient exactness.

While two hypotheses are possible the mind of the investigator is in a state of indecision. Nothing is more unlike scientific thought than to reject a hypothesis without proof that it is false.

(ii) Practical.—A state of indecision between alternative

[1] *Ibid.*, pp. 258-260.

courses is well known to us in practical life. Again, we have two hypotheses, and often we have to select one of them and act upon it.

A student of shortened means may need both a set of books and a new overcoat, but may have money enough for only one of them. So they become alternatives. If he buy the books, he must go cold or shabby in winter; if he buy the coat, his intellectual work will be hindered. He is in a dilemma. He is forced to choose between two courses, each of which is undesirable.

Except that the mind searching for truth neither likes nor dislikes the consequences of alternative hypotheses, the dilemma in practical life reproduces on a small scale the indecision between rival hypotheses which marks the intellectual dilemma.

In public affairs dilemmas of national conduct are usually known as 'crises.' On their right solution may depend the weal or woe of millions of people.

Elizabeth's vacillating conduct towards the Netherlands, then in revolt against Spain, often put that country into a dilemma, equally unhappy for itself and for England. In 1584 "St. Aldegonde, in reply to the message sent by Walsingham in November, had answered that the States were at the last extremity. They could not hold out beyond the following summer without help, and if England continued to keep aloof, there were but two alternatives before them. If the whole of the States, including Holland and Zealand, would consent to be annexed to France, the French . . . were . . . ready to risk a war for the acquisition; otherwise necessity was a law of iron, and they must submit to Spain.

It was hard to say which of these two results would be most unwelcome in England."[1]

When Mary of Scotland sought refuge in England, the

[1] Froude: *History of England*, ch. 66.

English Government found itself faced with several alternatives, none of which was to its taste. On June 20th, 1568, the Privy Council deliberated on what action should be taken. They could either restore Mary to her throne 'without trial heard,' or let her go where she would, or restore her in name but not in power, or detain her in England. To do the first "would be to declare her innocent of the crimes with which she was charged, and would enable her to crush and ruin the best friends that England possessed among her subjects. To let her go would be to throw her upon France; and 'her Majesty would never be free from practices and enterprises.' To restore her 'in title and name, without authority of government,' was thought 'so hard a matter,' that it would be even dangerous to proceed that way. She would 'burn with hate and revenge.' The French and the Pope would take up her cause; and after her breach of faith on the treaty of Leith, no promises which she might make could be relied upon."[1]

The decision to adopt the fourth alternative was abundantly shown, during the next twenty years, to have been by no means free from inconveniences to Elizabeth's Government.

(iii) Rhetorical.—It has always been a favourite device in argument for one disputant to 'impale' the other 'on the horns of a dilemma.' It was from consideration of such devices that Aristotle first analysed this form of thought.

In its nature it is not different from the more important cases we have been considering. Its characteristic is that it compels a choice of alternatives.

Junius wrote to the Duke of Grafton: "If you will not speak out, the public have a right to judge from appearances. We are authorised to conclude that you either differed from your colleagues, whose measures you still affect to defend, or that you thought the administration of the King's affairs no

[1] *Ibid.*, ch. 50.

longer tenable. You are at liberty to choose between the hypocrite and the coward."[1]

(iv) Rebutting Dilemmas.—Rhetorically a dilemma is often undeservedly effective. It has an air of clinching the matter which is peculiarly impressive to untrained minds. Yet, a little ingenuity will often upset it. Take the case of the poor student hesitating between books and overcoat. If he be pessimistic, he will put his case thus: 'If I buy the books I shall be cold and shabby; if I buy the coat I shall be unable to do my work.' As he can buy but one of them, he foresees an unhappy winter. But suppose him in an optimistic frame of mind. Then he puts it: 'If I buy the books I shall get on with my work; if I buy the coat I shall be comfortable. So, whichever I do, I shall profit.'

Compare these two equally conclusive dilemmas, and note that each simply negatives each consequent and transfers it to the other antecedent. This is called *formally rebutting a dilemma.*

A dilemma may be met in other ways. Macaulay thus deals with one advanced by James Mill as an argument in support of representative democratic government: "He says, that if men are not inclined to plunder each other, government is unnecessary, and that, if men are so inclined, kings and aristocracies will plunder the people. Now, this, we say, is a fallacy. That *some* men will plunder their neighbours if they can, is a sufficient reason for the existence of governments. But it is not demonstrated that kings and aristocracies will plunder the people, unless it be true that *all* men will plunder their neighbours if they can."[2] This is a much more adequate mode of answer. The basis of the alternative offered is shown to be unsound.

(v) Framework of Dilemmas.—Analysis of a dilemma shows that its framework is the conjunction of a compound

[1] *Let.* 36.

[2] *Westminster Reviewer's Defence of Mill*, in *Miscellaneous Writings.*

hypothetical proposition as major premise with a disjunctive proposition as minor premise, compelling a choice of alternatives, usually constructively, by asserting the alternative antecedents, but at times destructively, by denying alternatively the consequents.

The desired result may be attained more or less directly. Hence, the framework appears in various detailed forms. Our own experience makes it apparent also that the alternatives are not always limited to two, and the example of the relation of Elizabeth's Government to Mary of Scotland illustrates the same point. But however many they be, the same principles are involved. Of course, the name 'dilemma' is properly applicable only to those with two alternatives. Those with three are sometimes called 'trilemmas,' those with four 'tetralemmas,' and those with more than four 'polylemmas.' But these terms are not frequently used.

Analysis of the simpler forms—the dilemmas—shows the following frameworks to be possible, though trial will convince the reader that some are more easily used than others.

(1) *Simple Constructive.*

(*a*) *If either* **A** *or* **B**, *then* **C**,
Either **A** *or* **B**,

∴ **C.**

(*b*) *If either* **A** *or* **B**, *then either* **C** *or* **D**,
Either **A** *or* **B**,

∴ *Either* **C** *or* **D.**

(2) *Simple Destructive.*

(*a*) *If* **A**, *then both* **C** *and* **D**,
Either **not C** *or* **not D**,

∴ **Not A.**

(*b*) *If both* **A** *and* **B**, *then both* **C** *and* **D**,
Either **not C** *or* **not D**,

∴ *Either* **not A** *or* **not B.**

(3) *Complex Constructive.*

If **A**, *then* **C**, *and if* **B**, *then* **D**,
Either **A** *or* **B**,

∴ *Either* **C** *or* **D**.

(4) *Complex Destructive.*

If **A**, *then* **C**, *and if* **B**, *then* **D**,
Either **not C** *or* **not D**,

∴ *Either* **not A** *or* **not B**.

The second form of the simple constructive dilemma is simple because the alternative hypotheticals have only one consequent. The conclusion is disjunctive because this single consequent is disjunctive in form.

Similarly, the second form of the simple destructive dilemma is not complex, although it has a disjunctive conclusion, for that conclusion is merely the denial of the one single antecedent of the hypothetical.

It thus appears that these are only somewhat complex varieties of the simple forms.

CHAPTER XIII.

DEDUCTIVE APPLICATION.

1. Importance of Deductive Application.

General laws are suggested by the analysis of a few facts. They are established by the agreement of consequences inferred from them with a few other facts. So the outlines of systems are marked out. To fill in these systems, so that they approximate in richness to the reality they represent, is the last step the mind has to take; but it is one that has to be taken many times, and in many directions.

This was clearly seen by Aristotle, the father of deductive logic. He wrote: "There is the same course to be pursued in philosophy, and in every science or branch of knowledge. You must study facts. Experience alone can give you general principles on any subject. This is the case in astronomy, which is based on the observation of astronomical phenomena; and it is the case with every branch of science or art. When the facts in each branch are brought together, it will be the province of the logician to set out the demonstrations in a manner clear and fit for use. When the investigation into nature is complete, you will be able in some cases to exhibit a demonstration; in other cases you will have to say that demonstration is not attainable."[1]

Induction and deduction, then, are closely related. The former gives us principles; the latter applies them.

[1] *Prior Analytics*, i. 30 (trans. Grant: *Aristotle*, p. 66).

2. Examples.

(*a*) *The Mariner's Compass.*—Such application leads us now to fuller understanding of facts, now to more effective use of them. We owe the mariner's compass to deductive inference from an observed constant relation. "Flavio Gioja . . . found that if a needle or piece of iron which has been magnetised is hung by its middle from a piece of light string, it will always turn so that one end points to the north and the other to the south. He therefore took a piece of round card, and marking it with north, south, east, and west, he fastened a magnetised needle upon it pointing from north to south; he then fastened the card on a piece of cork and floated it in a basin of water. Whichever way he turned the basin the needle carried the card round till the N. of the needle pointed to the north, and the S. to the south, and from the other marks on the card he could then tell the direction of the west, north-west, etc."[1]

(*b*) *Aseptic Surgery.*—Early in his professional career, the late Lord Lister was impressed by the number of cases in which a successful surgical operation was followed by 'surgical fever,' often developing malignant symptoms, and ending fatally. The common hypothesis was that this fever was due to the decomposing of tissue by the action of the oxygen of the air.

Then Pasteur proved "that fermentation and putrefaction were not mere chemical changes, initiated and promoted by contact with the atmosphere, but that they were due to the activities of minute living organisms, and could not occur, even in the most fermentable or most putrescible liquids, if these organisms were excluded.

With genius kindred to that of Pasteur himself, Lister at once grasped the surgical possibilities of the discovery, and applied himself with steady determination to render them of practical applicability for the preservation of life

[1] Miss A. B. Buckley: *Op. cit.*, pp. 53-54

In those days, before the very beginnings of what is now called bacteriology, little or nothing had been learned about the organisms themselves, which were somewhat vaguely spoken of as 'germs,' and were supposed to be everywhere suspended in the atmosphere. Lister's first idea, in dealing with a wound, was to kill any germs which had already found admission to it, and then to protect it against the entrance of more. After much consideration, he selected carbolic acid as his germicide, and cotton wool, variously medicated, as his protective.

The cases first treated in this way were cases of compound fracture, that is to say, of broken bones in which the soft parts covering the bones were divided, and the fragments were exposed. The wounds thus made were thoroughly washed out with carbolic acid, which formed with effused blood a firm coagulum, and were covered with the protective dressing, which was well secured by a bandage. The patients remained free from 'surgical fever,' and when the wounds were opened, they were clean and sweet. The dressings were renewed with strict precautions against the admission of 'germs,' and the cases recovered without interruption.

Lister's first public announcement of his method was made in March, 1865. . . .

Improvements of method, as Lister successively introduced them, were all in the direction of increased carefulness attended by increased simplicity. . . .

The one thing to be dreaded was the introduction into a wound of any of the micro-organisms which are capable of producing 'septic' or putrefactive changes; and, whereas the new surgery had originally been described as 'anti-septic,' it soon became customary to call it 'aseptic' instead; with the idea of indicating that the object of the surgeon was rather to exclude certain organisms than to destroy them.

Finally, the matter reduced itself to this: that the skin in the neighbourhood of the part to be operated upon was

cleansed with what would seem to the uninstructed to be superfluous and exaggerated care; that the hands of the surgeon and his assistants were similarly treated, that all instruments and materials intended to be brought into contact with the patient were sterilised by boiling, and were lifted out of the boiling solution into a bactericide one, in which they remained immersed until the moment of use, and that all the dressings were sterile in themselves, and impervious to micro-organisms from without. . . .

Of the actual extent of the benefits conferred by Lister upon the world, no estimate can be other than conjectural. It might be possible to obtain hospital statistics, showing the enormous increase in the number of operations performed and the enormous diminution of the percentage rate of mortality attending them; but to do this would leave the still wider field of private surgery untouched. It was asserted in 1900, with much appearance of probability, that he had already saved more human lives than all the wars of the expiring century had sacrificed; and, since then, the complete adoption of his methods by the military surgeons of Japan has still further added to his trophies and enhanced his fame. It is unquestionable that, in regard to the cure of disease, the relief of suffering, and the saving or prolongation of life, he has been the greatest of all benefactors to the human race. 'My Lord,' said Mr. Bayard, the American Ambassador, in proposing his health at a Royal Society dinner, 'it is not a profession, it is not a nation, it is humanity itself which, with uncovered head, salutes you.'"[1]

These were increases of practical knowledge, and each has had far-reaching consequences. We will now illustrate the extension of theoretical knowledge in the explanation of facts, long familiar, but not understood.

[1] From Obituary Notice in *The Times*, 1912.

(c) *The Rainbow.*—"When the laws of the reflection and refraction of light had been established, a new and poignant exemplification of them was found in the explanation of the rainbow by the reflection and refraction of light in the spherical drops of a shower; and again, another, no less striking, when the intersecting luminous circles and mock suns, which are seen in cold seasons, were completely explained by the hexagonal crystals of ice which float in the upper regions of the atmosphere. The darkness of the space between the primary and secondary rainbow is another appearance which optical theory completely explains. And when we further include in our optical theory the doctrine of interferences, we find the explanation of other phenomena; for instance, the supernumerary rainbows which accompany the primary rainbow on its inner side, and the small halos which often surround the sun and moon."[1]

(d) *Gunpowder.*—The explanation of the explosive force of gunpowder is found in chemical laws. "The force is due to the sudden development of a large volume of nitrogen and carbonic acid gases, which at the ordinary temperature of the air would occupy a space equal to about three hundred times the bulk of the powder used, but from the intense heat developed at the moment of the explosion, the dilatation amounts to at least fifteen hundred times the volume of the gunpowder employed."[2]

(e) *Speed of Canal Boats.*—"Many . . . curious facts might be mentioned which when once noticed were explained as the effects of well-known laws. It was accidentally discovered that the navigation of canals of small depth could be facilitated by increasing the speed of the boats, the resistance being actually reduced by this increase of speed, which enables the boat to ride as it were upon its own forced wave. Now, mathematical theory might have

[1] Whewell: *Novum Organon Renovatum*, p. 237.
[2] *Ibid.*, p. 239, note.

predicted this result had the right application of the formulæ occurred to any one."[1]

So, deduction is never exhausted. "The possession of a true theory does not by any means imply the foreseeing of all the results. The effects of even a few simple laws may be manifold, and some of the most curious and useful effects may remain undetected until accidental observation brings them to our notice."[2]

Deduction can do more than explain known facts. It can establish the reality of facts "which have not been, and in many cases probably never can be, verified by trial."[3] For, "deduction is certain and infallible, in the sense that each step in deductive reasoning will lead us to some result, as certain as the law itself."[4]

(*f*) *Tides of the Moon.*—For example, "There was a time when the materials of the moon were so hot as to be soft and yielding, and in that soft and yielding mass the attraction of our earth excited great tides. We have no historical record of these tides (they were long anterior to the existence of telescopes; they were probably long anterior to the existence of the human race), but we know that these tides once existed by the work they have accomplished, and that work is seen to-day in the constant face which the moon turns towards the earth. . . .

That the moon should bend the same face to the earth depends immediately upon the condition that the moon shall rotate on its axis in precisely the same period as that which it requires to revolve round the earth. The tides were a regulating power of the most unremitting efficiency to ensure that this condition should be observed. If the moon rotated more slowly than it ought, then the great lava tides would drag the moon round faster and faster, until it attained the desired velocity; and then, but not till then, they would

[1] Jevons: *The Principles of Science*, pp. 535-536. [2] *Ibid.*, p. 536.
[3] *Ibid.*, p. 548. [4] *Ibid.*, p. 534.

give the moon peace. Or; if the moon were to rotate faster on its axis than on its orbit, again the tides would come furiously into play; but this time they would be engaged in retarding the moon's rotation, until they had reduced the speed of the moon to one rotation for each revolution."[1]

(*g*) *Ice under Pressure.*—In the middle of the last century "Professor James Thomson deduced from Carnot's theory of heat that the application of pressure would lower the melting-point of ice. He even ventured to assign the amount of this effect, and his statement was afterwards verified by" his brother William, afterwards Lord Kelvin.[2]

The theory thus established explains how it is that two blocks of ice laid side by side in contact gradually fuse into one. The pressure at the point of contact melts the ice; the water thus formed escapes, and freezes again into ice. The motion of glaciers finds explanation in this process.

In it, too, is found the explanation of the possibility of skating on ice. It is not a matter of hardness or smoothness of surface, for in both these respects, polished steel, glass, and many other substances surpass ice, even when it is newly formed on a still pond. Yet one cannot skate on any of them, while skating is possible even on much-worn ice.

It can easily be shown by experiment that a weight will slip down a slab of ice inclined at a much smaller angle than is needed with a sheet of glass, and that the heavier the weight the less the ice needs be inclined. This puts the cause of the slipping in a relation between the nature of ice and the pressure upon it.

Here is a link of connexion with the laws enounced by James Thomson. Bringing the case under those laws, it is deduced that melting and regelation go on continuously under the blade of a skate pressed on to the ice by the full weight of the skater. The water formed acts as a lubricant

[1] Sir R. Ball: *The Story of the Heavens*, pp. 530-531.

[2] Jevons: *Ibid.*, p. 542.

in reducing the friction between the blade of the skate and the ice. As both the melting and the regelation are instantaneous, the facts cannot be directly observed. They are the inevitable consequences of the laws of the relation of pressure and freezing-point of ice, and they are indirectly shown to be real by the explanation they give of the anomalous fact that skating is possible on rough ice, but not on any other material, no matter how smooth and hard a surface it may possess.

3. Analysis of Deductive Inference.

(i) Meaning of Syllogism.—Such examples make clear that deduction is a valuable instrument in both the theoretical and the practical spheres. Indeed, we are always drawing conclusions from general truths, principles, and rules. The most enthusiastic investigator into new realms of knowledge daily applies in thought and conduct the accumulated wisdom of the past.

We all know that at times we find that wisdom foolishness, at others we misapprehend its bearing, yet again we deduce from it consequences it does not involve. No mode of inference is free from the possibility of fallacy, but familiarity with the nature of the reasoning is the best safeguard against it. We have already considered the errors that result from taking as true principles, what are not true principles,[1] and from applying rules to cases they do not fit.[2] We must now bring to light the principles which are operative in valid deductive reasoning.

Such an analysis was first made by Aristotle. He approached the subject through a comparative examination of the deductive arguments used in dialectic. These he reduced to their simplest forms, and was then able to lay down the conditions on which they were valid. To all such forms he

[1] Cf. pp. 119-122. [2] Cf. pp. 132-133.

gave the general name of *Syllogism,* and his definition of 'syllogism' was both simple and comprehensive: "*an inference in which a new result follows necessarily from certain presuppositions.*"[1] That is to say, if one accepts as true the premises of a valid syllogism, one is bound in consistency to accept the conclusion.

One such form, in which the premises are a hypothetical and a categorical proposition, we examined in the last chapter.[2] We have also seen that the denial of all the alternatives but one of a disjunctive proposition implies the assertion of that one. Such a form of argument satisfies the definition, and is known as a 'mixed disjunctive syllogism.' Each of these mixed forms is simple, and either can be reduced to the other.[3]

When both the premises are of the same kind, the syllogism is called 'Pure.' Various forms are possible. These we shall examine in the next chapter. We are now concerned with the principles common to them all.

Aristotle worked out the doctrine in the simplest case; that in which premises and conclusion are all categorical propositions. The principles thus found are equally true of syllogisms in which they are all hypothetical or disjunctive propositions, for the validity of the inference depends wholly on the correct drawing out of what the bare combination of the premises involves.

Syllogism, then, is the expression in the simplest form of the results yielded by analysis of deductive inference.

(ii) THE MIDDLE TERM.

(*a*) *Function.*—What, then, is this simplest form? One in which the union of two premises gives a certain conclusion, and the premises are categorical propositions expressed in the normal forms.

Its most general principle is known as the *Dictum de omni*

[1] *Prior Anal.*, i. 1. [2] Cf. pp. 225-228. [3] Cf. p. 46.

et nullo: 'Whatever is affirmed or denied universally of any class can be affirmed or denied of any part of that class.'

Two such propositions between them have four terms. That the premises may be united, one of these terms must be common to both. This link between the two premises is called the *Middle Term.* Its one function is to secure that the two premises are really bound together. When it has fulfilled that function it can pass out of consideration, and the conclusion asserts a relation between the two remaining terms, sometimes called the *Extreme Terms.* That which appears in the conclusion as subject is called the *Minor Term,* and the premise in which it is found, the *Minor Premise,* while the predicate of the conclusion is the *Major Term* and the premise in which it occurs the *Major Premise.*

The most direct symbolic statement of this is: *S is M, and M is P; therefore, S is P.* Evidently, *M* must have identically the same reference in both premises, or no connexion is made between them.

(*b*) *Ambiguity.*—If *M* is expressed by an ambiguous word or phrase, having different meanings in the two premises, the first condition of syllogistic reasoning is broken. The four terms in the premises are then all distinct; there is no bond of union. Such a 'logical quadruped' cannot take a step forwards without floundering.

We have already pointed out how prone thinkers and writers on human life and actions are to be misled by the ambiguity of such terms as 'happiness,' 'nature,' 'wealth.'[1] 'Conformity to nature' is a particularly heinous and frequent offender, referring now to the spiritual, now to the animal, nature of man, and shifting without warning from the one meaning to the other. For example, from the proposition that as education trains a child's nature, it must be in harmony with that nature, it has been deduced that it should

[1] Cf. pp. 181; 33-34.

not check any youthful impulse. But, it is the spiritual nature which education has to foster, while childish impulses are largely the expression of that animal nature which it is part of the work of the spiritual nature to hold in subjection. The supposed argument has, therefore, no force. The two propositions are not connected by a common term, and so are not premises yielding a conclusion.

(*c*) *Distribution.*—The middle term may be ambiguous in its reference, even when definite in its meaning. Different parts of its denotation may be referred to in the two premises. From 'stealing is an offence against honesty' and 'slander is an offence against honesty' the conclusion that 'slander is stealing' cannot be drawn. If, however, in one or both of the premises, the middle term is used in its whole reference, such ambiguity of reference is impossible. For if in one premise explicit reference is made to *every M*, the *M's* of the other premise must be among them, and it is this common part which is the bond of connexion.

Against the use of terms ambiguous in meaning no formal safeguard can be given. It is an individual matter, depending on powers of clear thinking and precise speaking.

But against ambiguity in denotative reference, the formal rule that the middle term must be distributed in at least one premise is wholly effective.

(iii) Quality of Premises.—A true syllogism so binds together two premises by a common point of reference that a relation between the other two terms is implied. It follows from this that both premises cannot be negative. The exclusion of two terms from the scope of a third cannot justify relating them together. Because neither sponges nor sea-anemones are plants, we cannot infer that either is a species of the other.

Of course, the negation must be real, and not merely rhetorical. From 'whatever tax is not unequal in incidence is not unjust' and 'a heavy income-tax is not unequal in

incidence' we can conclude that 'a heavy income-tax is not unjust.' But that is because the minor premise *affirms* that its subject comes under the rule stated by the major. 'Tax-not-unequal-in-incidence' is the middle term, common to both premises. So the minor premise only seems negative at a first glance; the apparent ambiguity in the middle term is not difficult to detect and set right.

If negation is in a syllogism at all, it must be both in one of the premises and in the conclusion. When one of the extreme terms is shut out from the middle term, and the other extreme term wholly or partially identified with it, it is evident that the extreme terms are excluded from each other, and that they can be so separated on no other condition. For, if both extreme terms be wholly or partially identified with the middle term, they cannot on that ground be denied of each other. So a negative conclusion cannot follow from two affirmative premises.

(iv) DISTRIBUTION OF EXTREME TERMS.—The whole force of a syllogism is, that acceptance of the premises compels acceptance of the conclusion. Evidently, then, the conclusion must not make an assertion in any way wider than that in the premises. Expressed formally, this gives the rule that unless an extreme term is distributed in its premise it must not be distributed in the conclusion. To infer from 'All unequal taxation produces discontent' and 'A heavy income-tax is not unequal' that such a tax will not provoke discontent is quite illegitimate. Inequality of incidence is not the only aspect of a tax capable of raising the wrath of the tax-payer. The transgression of this principle is known as *Illicit Process* of the major or minor term as the case may be.

(v) SUMMARY OF SYLLOGISTIC PRINCIPLES.—We may sum up the conditions which every valid syllogism fulfils.

(1, 2) Two premises lead to a conclusion when one of their four terms is common to both. Hence, *a syllogism must contain* (1) *Three propositions*, (2) *Three terms.*

(3) *This 'middle' term must be distributed in at least one of the premises.*

(4) *A term can be distributed in the conclusion only when it is distributed in its premise.*

(5) *A negative premise and a negative conclusion mutually involve each other.*

(6) *No conclusion can be drawn from two negative propositions.*

(vi) Corollaries.—From these principles several subordinate rules may be deduced. For as the middle term must be distributed and occurs only in the premises, and as any term distributed in the conclusion must also be distributed in its premise, there must always be one more distributed term in the premises than in the conclusion. Hence—

(1) *If one premise is particular, the conclusion must be particular.* For, if both premises are affirmative they then can only distribute one term: while a universal conclusion would require the distribution of the minor, as well as of the middle, term;

and, if one premise is negative and the other affirmative, they can then distribute between them only two terms; while a universal negative conclusion would require that both major and minor, as well as middle, terms be distributed.

(2) *One premise must always be universal.* For, if both are particular affirmative, they distribute no term;

and, if one is particular negative and the other particular affirmative, they distribute only one term between them; while as any conclusion must be negative, it would require the distribution of the major, as well as of the middle, term.

(3) *If the minor premise is negative the major premise must be universal.* For, the conclusion must be negative, and so require the distribution of the major term;

while the major premise must be affirmative, and can only distribute the major term when it is universal and has that term as its subject.

4. Objections to Syllogistic Analysis.

(i) Purpose of the Analysis.—Deductive inferences follow each other in our minds in a long chain. Now, the strength of a chain depends on that of its links. Syllogism is a test of each link. If we commit fallacy it is in a definite spot. Syllogism puts its finger on that spot. It does this by setting forth the naked framework of valid deductive inferences. It is, as Leibniz said, "a kind of universal mathematics."[1]

(ii) Its Utility.—A clear grasp of this enables us to meet an objection sometimes urged. We are told that syllogism has ceased to be of use, because, whatever the mediaeval philosophers may have done, people do not reason in syllogisms nowadays.

If it be meant that arguments are seldom expressed in a bald syllogistic form, the objection is true in fact, but quite beside the point. The doctrine of syllogism is logical, not rhetorical. It is concerned with relations of thought, not with the verbal form in which thought clothes itself. Syllogism is an analytic symbolic framework, stripped bare of all ornament, dealing with nothing but the skeleton of thought. Such a framework upholds every valid deductive inference. If, in laying bare the framework, we find it disjointed and rickety, the argument is worthless, no matter how attractively rhetoric may have bedizened it.

It is true, too, that our thought frequently proceeds inductively. We note facts and seek an explanation of them. This is characteristic of the arguments we find in modern books, as well as of those we ourselves use in matters of daily life. Thought is less preponderatingly deductive in our day

[1] *Nouveaux Essais*, iv. 17, § 4.

than it was when men accepted without examination a number of general assumptions in relation to every realm of experience.

Nevertheless, as we have seen, induction is the verification of guesses, and that implies the deduction of the consequences of our guesses and the comparison of them with fact. Moreover, all induction has for its aim the establishment of wide general truths, from which safe deductions may be made. So, as knowledge becomes more perfect, deduction becomes proportionately more important. In astronomy, for example, deduction looms large.

In every valid deductive inference, syllogistic principles are inherent. If one is violated, the whole chain of reasoning is worthless, for the strength of a chain is that of its weakest link. He deduces best in whose mind the canons of syllogism have become operative principles, so familiar in use that their presence is not consciously recognised.

The fact that less use is made of deduction than formerly is, therefore, a reason for giving special attention to its nature and rules. We must acquire deliberately what we do not pick up unconsciously in daily practice, and from the example of books and men. For life compels us to reason deductively, and that often, and in matters of the highest importance, such as questions of right and wrong. Power of accurate deductive reasoning is essential to the wise guidance of life.

If, however, the objection mean that most people, as a matter of fact, have ceased to govern their reasonings by syllogistic principles, and that, therefore, syllogistic principles are no longer worthy of attention, we can but retort that such an 'argument' shows the need of syllogism. It affirms that, because many people reason badly, the principles of sound reasoning should be relegated to the lumber-room of an intellectual museum of antiquities. As well contend that because many people do evil deeds the moral law should be

regarded as obsolete, or because many people fall ill medicine should no longer be studied.

No one can deny that many people *do* reason loosely; that they are more influenced by showy bombastic rhetoric—misnamed eloquence—than by calm and rigid reasoning; that, in consequence, they are incompetent to guide themselves or others in matters in which strict inference is demanded. But the recognition of the prevalence of a fault is no reason for the abolition of the remedy. In short, while men live they must reason deductively; all such reasoning is valid only when it is in conformity with syllogistic principles; therefore, familiar acquaintance with those principles is worth acquiring.

(iii) Its Validity.—Deductive reasoning is essentially the drawing out the consequences of a general principle. We have seen that a general principle is established inductively by an analysis which lays bare the relations that hold facts together. Our whole process of thought rests on the assumption that relations are constant. That is an ultimate postulate, without which thought could not even begin.

The major premise, which states the principle, is, therefore, not a summing up of an enumeration of instances.[1] If it were, no advance in knowledge could be made by deductive inference. Indeed, it would be true to say that deductive inference would be impossible, for such inference reaches results justified by the premises, but not explicitly stated in them. The drawing out of results is always going on in new directions. A principle is established whenever a relation is truly found. It is because we must postulate that relation as constant that we can lay it down as a general proposition.

We may express the syllogism in various ways. We may say: '*P* may be predicated of *M*, and *S* is contained under *M*;

[1] Cf. pp. 166-167.

therefore, P may be predicated of S.' This most directly exhibits the mode of thought. But, as every term has both connotation and denotation, we are justified in writing the major premise 'Every M is P'; for, if the relation between M and P is a universal one, P may be truly predicated of every M.

This denotative form has the advantage of making explicit the distribution of each term, and so guarding against formal fallacy. But, taken by itself, it suggests that it simply sums up a counting of instances of M If this were so, the syllogism would beg the question. For, if the enumeration of M's is complete, and S is an M, then S must be known to be P before we can say 'Every M is P.'

This objection has, indeed, been brought against the syllogism. If it were well founded, it would destroy, not only the validity of the syllogistic analysis of deductive reasoning, but the possibility of all inference. For it assumes that a general proposition can be nothing but a summary of a counting of instances. If this were true, we could never pass beyond the range of the cases examined. Thus, both induction and deduction would be impossible. In short, man could not reason; he could only count. The assumption that this *is* the origin of the major premise of a syllogism is itself as frank a begging of the question as it is possible to conceive.

It requires little thought to show that the contention that the conclusion of a syllogism must be known before the major premise can be legitimately enounced is unsound. When a law is passed, the cases which will be decided under it are certainly unknown, for they are all in the future. Every legal decision is the conclusion of a deductive inference, in which the law is the major premise, and the case brought to trial the minor premise. The decision of the court is the conclusion. Many towns during war-time have been under strict prohibition of the exposure of lights; many inhabitants have been brought under the regulations; the practical con-

clusion of a fine has shown the force of the syllogism. No sane person could contend that all the offenders had to be foreseen and counted before a regulation of the form 'All who show unshaded lights are liable to a fine' could be promulgated.

Nor can it be maintained that all the applications of scientific laws must be known before the law can be stated. The application to skating of the laws of the relation of pressure to the freezing-point of water was assuredly not considered before those laws were established as certainly true.

Advances in knowledge are continually made by deductive reasoning which can be analysed into syllogisms. They are advances, because those cases of the operation of the general laws were not foreseen; they are knowledge, because they are seen to be necessitated by the premises as soon as the syllogism is constructed.

It is the construction of the syllogism that takes the time, and taxes the imagination. Consider the example of skating. As soon as it is seen to be an instance of the general relation of pressure to freezing-point of water, it is explained. But for long it had not been seen. The law was known, the fact was known; but thought had not connected them. And the connexion is through the middle term. Thus, the formation of syllogisms is the finding of appropriate middle terms. When they are found the particular cases are brought under the general relations which explain them. They find their proper places in the system.

5. Scope of Deductive Inference.

When we study the world of physical fact we demand that hypotheses shall be both suggested and confirmed by such facts. We say our thought is true when the relations we have imagined are proved to hold in the world of things.

Our thoughts, however, are not limited to facts. We can

appreciate relations of goodness, truth, beauty, more perfect than those we find fulfilled in daily life.

The imagination of the mathematician conceives a world of abstractions "in which it is possible to draw through a given point an indefinite number of straight lines which do not intersect a given straight line, each of them having a common perpendicular with the given straight line; in which similar non-congruent triangles are impossible, and a straight line can be drawn through a given point parallel to two intersecting straight lines."[1]

Philosophers and poets have constructed utopias in which men of nobler endowments of character and intellect live in a more perfect material and social environment than any known on earth.

Swift, from the hypothesis of a race of giants, worked out the kingdom of Brobdingnag; from that of a nation of dwarfs he constructed Lilliput. Yet more removed from every-day fact were the consequences which followed from the supposition of a race of horses, intellectually and morally the superiors of men, and consequently holding men in subjection.

Shakespeare imagines a Lear or a Hamlet, placed in equally imaginary circumstances; and none but the veriest Gradgrind would deny that he moves our souls by the truth of his presentment. What is this truth? It is conformity less with the facts of the physical world than with the ultimate relations of the moral world.

These are even less perfectly fulfilled in men's relations with each other than are the abstract laws of physics in the cruder relations of things. For, as human nature is more complex than inorganic nature, its relations are more obscured by mutual interference. Yet, when some of them are set before us embodied in imaginary beings, sufficiently like our-

[1] Prof. A. H. Leahy in *Journal of Experimental Pedagogy*, vol. iii., p. 388.

selves to evoke our sympathy, so as to make them stand out clearly, we feel a responsive echo in our souls. We recognise them as elements in a truer picture of what the relations of man to man should be than is offered by the society in which we live.

So, generally, in a novel, a poem, or other work of imagination, the truth demanded is truth within the universe of discourse—the relevant realm of existence. The constituent elements are real only to fancy, but our dreams prove to us that at the time that is a reality unquestioned by us. When we are awake the control of thought, which in dreams is in abeyance, is operative. So we demand a consistency which in dreams is absent without surprising us. We are willing to grant any initial hypothesis—as Swift's imagination of houyhnhnms and yahoos—but we demand that all that is deduced from it shall be a legitimate consequence of it. When any part of the structure fails to fulfil the requirement, either the initial hypothesis is overthrown or this contradictory element must be modified or explained away.

Thus, analysis of the principles involved in the process of thought yields the same results whether the matter be physical fact or human imaginative thought.

CHAPTER XIV.

FORMS OF SYLLOGISM.

1. Figures.

The analysis of deductive reasoning into the bare form of syllogism lays stress on relations of inclusion and exclusion between classes. This is the most convenient way in which to examine the formal relations of propositions expressed in normal forms. It is an interpretation of the primary meaning of a proposition, that something is asserted of a subject. For, we may regard whatever we assert as the name of a class marked out by the possession of that characteristic.[1]

In the premises of a syllogism there are three terms, one of which occurs twice. If we analyse the possible relations of extent of these, we find them to be—

(1) when the middle term is intermediate in width between the two extremes;

(2) when it is wider than both the others;

(3) when it is narrower than both the others.

In every proposition the predicate is the wider term. Thus, these three cases are expressed by three arrangements of terms, differing in the position of the middle term.

Symbolising the middle term by M, the major term by P, the minor term by S, and writing the major premise first, these are represented by—

(1)	M $\quad$ P		(2)	P $\quad$ M		(3)	M $\quad$ P
	S $\quad$ M			S $\quad$ M			M $\quad$ S

In each case the conclusion drawn is $S \quad P$.

[1] Cf. p. 76.

These arrangements are called the *First*, *Second*, and *Third*, *Figures* of syllogism respectively. They are based on real distinctions of relation, and so give appropriate forms in which to express inferences concerning such relations. On this ground they were the only figures recognised by Aristotle.

But, if in the first relation we wish for any reason to affirm or deny the narrowest term of the widest, instead of drawing the more natural conclusion of predicating the widest of the narrowest, we shall use the one remaining possible arrangement of terms—

$$\begin{array}{cc} P & M \\ M & S \end{array}$$

This is a transposition of the order of the First Figure. The natural conclusion from the premises would be of the form $P \quad S$. Instead of this we draw the converse, $S \quad P$, as conclusion. Though, then, this *Fourth Figure* is formally valid, yet it is an excrescence on the doctrine of syllogism, regarded as an analysis of the processes of thought.

2. Moods.

(i) Determination of Moods.—These empty schemes of figures give no indication of the quality and quantity of their constituent propositions. They only show which term is subject, and which predicate, in each of the premises and in the conclusion. They become schemes of syllogism when the empty relations are filled out by propositions of the normal types.

But not every permutation of the four forms **A**, **E**, **I**, **O**, is a valid syllogism. We cannot, for example, have two negative premises, nor derive an affirmative conclusion from premises of which one is negative. In other cases, a combination is valid in one figure but not in another, as the different positions of M lead to different results when the

general rules of syllogism are applied. A valid combination is known as a *Mood*.

We could, of course, write down all the sixty-four possible mathematical permutations of **A**, **E**, **I**, **O**, in the four figures, and strike out those that offend. But the process is tedious, and does not give a positive reason why a mood which is saved is valid.

The determination may also be made by asking what combinations of premises will legitimately yield as conclusion each of the four propositions **A**, **E**, **I**, **O**, with S as subject and P as predicate, and finding the answer in the general rules of syllogism.[1] This is a valuable exercise, which the reader will do well to work out.

We will approach the problem from the side of the figures, working out the special characteristics, or rules, of each figure, and from them deducing the valid moods. Of course, whatever method is pursued, identically the same determination of moods results.

(ii) THE FIRST FIGURE.—From the distribution of terms it follows that—

$$\begin{array}{ccc} & M & P \\ & S & M \\ \hline \therefore & S & P \end{array}$$

(*a*) If one premise is negative it should be the major to secure the distribution of P;

(*b*) therefore, *the minor premise is affirmative*, and, consequently, does not distribute M;

(*c*) therefore, *the major premise is universal*, to secure the distribution of M.

(*d*) The quantity of the conclusion is that of the minor premise, because S is subject in both.

These considerations show that, the major premise is **A** or **E**, the minor premise **A** or **I**.

[1] See pp. 253-254.

Combining each minor with each major—

when premises are	**A A**, conclusion is	**A**.
„ „	**E A**, „ „	**E**.
„ „	**A I**, „ „	**I**.
„ „	**E I**, „ „	**O**.

Hence, there are *four* moods—**AAA**, **EAE**, **AII**, **EIO**.

(iii) THE SECOND FIGURE.

$$\begin{array}{cc} P & M \\ S & M \\ \hline \therefore S & P \end{array}$$

(*a*) As M is predicate in both premises, *one premise must be negative*, to secure the distribution of M;

(*b*) therefore, the conclusion is negative, and distributes P;

(*c*) therefore, *the major premise is universal*, to secure the distribution of P.

(*d*) The quantity of the conclusion is that of the minor premise, because S is subject in both.

These considerations show that the major premise is **E** or **A**, and the corresponding minors **A** and **I**, **E** and **O**, respectively.

Combining these—

when premises are	**E A** or **A E**, conclusion is	**E**.
„ „	**E I** or **A O**, „	**O**.

Hence there are four moods—**EAE**, **AEE**, **EIO**, **AOO**.

(iv) THE THIRD FIGURE.

$$\begin{array}{cc} M & P \\ M & S \\ \hline \therefore S & P \end{array}$$

(*a*) If one premise is negative it must be the major, to secure the distribution of P;

(*b*) therefore, *the minor premise is affirmative*, and does not distribute S;

(*c*) therefore, *the conclusion is always particular*.

These considerations show that, the major premise may be **A**, **E**, **I**, or **O**; and the minor **A** or **I**, but only **A** can be combined with **I** and **O**.

Combining—

when premises are **A A**, **A I**, or **I A**, conclusion is **I**.
" " **E A**, **E I**, or **O A**, " **O**.

Hence, there are *six* moods—**AAI**, **AII**, **IAI**, **EAO**, **EIO**, **OAO**.

(v) The Fourth Figure.

$$\begin{array}{ll} P & M \\ M & S \\ \hline \therefore S & P \end{array}$$

(*a*) *If either premise is negative, the major premise must be universal*, to secure the distribution of P.

(*b*) *If the major premise is affirmative, the minor must be universal*, to secure the distribution of M.

(*c*) *If the minor premise is affirmative, the conclusion is particular*, for S is undistributed in its premise.

These considerations show that the possible combinations are—

when premises are **A A** or **I A**, conclusion is **I**.
" " **A E**, " **E**.
" " **E A** or **E I**, " **O**.

Hence, there are *five* moods—**AAI**, **IAI**, **AEE**, **EAO**, **EIO**.

(vi) Subaltern Moods.—It may be noted that, as it is always *allowable* to state less than one is entitled to deduce, each mood with a universal conclusion has a valid—but useless—*Subaltern Mood*, in which the subaltern particular proposition is substituted for the legitimate universal as the conclusion—**I** for **A**; **O** for **E**. Thus, there are two subaltern moods in the first figure—**AAI** and **EAO**; two in the second figure—**EAO** and **AEO**; and one in the fourth figure—**AEO**. When these conclusions are drawn, the syllogism is said to be *weakened*.

(vii) Strengthened Syllogisms.—On the other hand, when M is distributed in both premises—in **AAI** in the

third figure, and **EAO** in the third and fourth figures—or when *P* is distributed in its premise, but not in the conclusion—**AAI** in the fourth figure—the syllogism is said to be *strengthened*, as the premises distribute more terms than is necessary.

(viii) Pure Hypothetical Syllogisms.—As hypothetical propositions can have the same distinctions of quality and quantity as categorical propositions, syllogisms, corresponding to all the valid moods, can be constructed in which each of the three constituents is a hypothetical proposition.

The particular hypothetical propositions are, however, equally well expressed in categorical form: 'If he catches typhus fever he may die' is only an alternative expression of the categorical judgement 'Some cases of typhus fever are fatal.' It follows that, a pure hypothetical syllogism is of any importance only when all its constituent propositions are universal. As with categorical syllogisms, the most valuable are in the first figure, *e.g.*—

If **Y** *is true, always* **Z** *is true;*
If **X** *is true, always* **Y** *is true;*

∴ *If* **X** *is true, always* **Z** *is true.*

This corresponds with **AAA**, in the first figure.

By substituting 'never' for 'always' in major premise and conclusion, we get a pure hypothetical syllogism corresponding to **EAE** in the first figure.

(ix) Pure Disjunctive Syllogisms.—As all disjunctive propositions are affirmative,[1] only affirmative moods are possible when all the constituent propositions are disjunctive. Moreover, we can only get a distributed middle term when one of the alternatives in the minor premise is the negative of one of those in the major premise. But such syllogisms are seldom used and are of no importance.

[1] Cf. p. 75.

3. Reduction.

(i) Purpose.—Aristotle taught that the validity of syllogisms in any figure but the first should be made manifest by 'reducing' them to that figure. Certainly, the first figure best lays bare the structure of deductive reasoning, of which the purpose is to bring particular cases under general rules; and the possibility of expressing all valid syllogisms in that figure shows that, in all its many forms, syllogism involves identical principles. Still, this does not imply that reduction to the first figure is necessary to the cogency of a syllogism. Reduction merely changes the form in which a syllogism is expressed; it adds nothing to its force.

(ii) Methods.

(*a*) *The Mnemonic Lines.*—Some inventive mediaeval genius wrote a set of doggerel lines, in which the processes were symbolised by letters, to help students to remember how to reduce the various moods. The words thus formed have become traditional names of the moods. The lines and their interpretation are—

> *Barbără, Cēlārent, Dărĭi, Fĕrĭō*que prioris:
> *Cēsărĕ, Cāmēstres, Festīnŏ, Bărōcŏ*, secundæ:
> Tertia, *Dārapti, Dĭsămis, Dātīsĭ, Fĕlapton,*
> *Bōcardō, Fērīsŏn*, habet: quarta insuper addit
> *Brāmantip, Cămĕnes, Dĭmăris, Fēsāpŏ, Frĕsīson.*

The vowels give the mood by indicating the form of proposition. *l*, *n*, *r*, *t*, also *b* and *d* when not initial, are without meaning.

In figures other than the first, directions for reduction are—

> *s*—convert preceding proposition *simply*.
> *p*—convert preceding proposition *per accidens*.
> *m* (*metathesis*)—transpose the premises.
> *c* (*conversio syllogismi*)—reduce indirectly.
> *B, C, D, F*—reduce to mood with same initial in first figure.

We will now give several illustrations of these processes—

(*b*) *Direct Reduction.*

(1) **AEE** in second figure (*Camestres*)
reduces to *Celarent*;
m—transpose the premises;
s—simply convert minor premise and conclusion.

Camestres.	*Celarent.*
Every P is M	*No M is S*
No S is M	*Every P is M*
∴ *No S is P*	∴ *No P is S*
	(conv.) *No S is P*

(2) **EAO** in third figure (*Felapton*)
reduces to *Ferio*;
p—convert minor premise *per accidens*.

Felapton.	*Ferio.*
No M is P	*No M is P*
Every M is S	*Some S is M*
∴ *Some S is not P*	∴ *Some S is not P*

(3) **AAI** in fourth figure (*Bramantip*)
reduces to *Barbara*;
m—transpose the premises;
p—convert conclusion *per accidens*.

Bramantip.	*Barbara.*
Every P is M	*Every M is S*
Every M is S	*Every P is M*
∴ *Some S is P*	∴ *Every P is S*
	(conv.) *Some S is P*

(*c*) *Indirect Reduction.*—The moods **AOO** in the second figure (*Baroco*), and **OAO** in the third figure (*Bocardo*), cannot be thus reduced, so an indirect method was applied to them. Thus—

Granting the premises, does the conclusion follow from them?

Now, if a proposition is false its contradictory is true; therefore, if the **O** conclusion be false, the corresponding **A** is true.

Assuming this, and combining it with the **A** premise, we have in each case a syllogism in *Barbara*—

Baroco.		*Barbara.*
Every P is M	————————	*Every P is M*
Some S is not M	(contr. of concl.)	*Every S is P*
∴ *Some S is not P*		∴ *Every S is M*

Bocardo.		*Barbara.*
Some M is not P	(contr. of concl.)	*Every S is P*
Every M is S	————————	*Every M is S*
∴ *Some S is not P*		∴ *Every M is P*

In each case the conclusion is the contradictory of the unused premise.

But the original premises are accepted as true; therefore, the new conclusion is false.

But it is validly deduced from its premises; therefore, one of those premises is false.

But one of them is one of the original premises; therefore, the other is false.

But this is the contradictory of the original conclusion; therefore, that conclusion is true; *i.e.* it is consistent with its premises.

This process is also called *Reductio ad absurdum* and *Reductio per impossibile.* It is applicable to any mood.

Such an indirect form of proof is often used by Euclid in

his demonstrations of geometrical relations, and is an effective weapon in controversy.

(*d*) *Direct Reduction by Obversion.*—Recently logicians have shown that *Baroco* and *Bocardo* can be reduced directly, though not to *Barbara*, by the use of obversion and contraposition. The mnemonic words *Faksoko* (*Baroco*) and *Doksamosk* (*Bocardo*) have been suggested to direct these processes, but they have not come into general use as names of the moods. In these the initial letter indicates to which mood in the first figure each is to be reduced, and *k* signifies obversion. The other letters point to the same processes as in the ordinary mnemonics. So, *ks* directs us to contraposit, and *sk* to obvert the simple converse. Thus—

(1) *Faksoko*

reduces to *Ferio*;
ks—contraposit the major premise;
k—obvert the minor premise.

Faksoko (*Baroco*).		*Ferio.*
Every P is M	(contrap.)	*No not-M is P*
Some S is not M	(obverse)	*Some S is not-M*
∴ *Some S is not P*		∴ *Some S is not P*

(2) *Doksamosk*

reduces to *Darii*;
m—transpose the premises;
ks—contraposit the major premise;
sk—obvert the converse of the conclusion.

Doksamosk (*Bocardo*)		*Darii.*
Some M is not P	(minor prem.)	*Every M is S*
Every M is S	(contrap. of major)	*Some not-P is M*
∴ *Some S is not P*		∴ *Some not-P is S*
	(conv.)	*Some S is not-P*
	(obv.)	*Some S is not P*

(iii) Reduction of Pure Hypothetical Syllogisms.—The processes of reduction can be applied to pure hypothetical syllogisms, but the reductions are mainly exercises in manipulation of symbols. We will leave the reader to work them out, giving the direct reduction by obversion of **AOO** in the second figure (*Faksoko* or *Baroco*) as an example :

If **Z**, *then always* **Y**	(contrap.)	*If* **not-Y**, *then never* **Z**
If **X**, *then sometimes not* **Y**	(obv.)	*If* **X**, *then sometimes* **not-Y**
∴ *If* **X**, *then sometimes not* **Z**		∴ *If* **X**, *then sometimes not* **Z**

4. Combined Syllogisms.

(i) Enthymemes.—Syllogism is the formal analysis of what is involved in deductive inference, in which a particular case is brought under a general rule. It, therefore, sets out explicitly all that is involved in such a form of argument. But in actual thinking we commonly assume one of the propositions in our own minds, and leave it to be understood by hearer or reader.

Thus, we might say 'The people are discontented because the taxation is unjust,' where we imply the major premise 'All unjust taxation causes discontent.'

Or we might omit the minor premise, and say 'No wonder the people are discontented, for all unjust taxation provokes discontent.'

Or, finally, we might say 'The taxation is unjust, and such taxation inevitably leads to discontent,' leaving the conclusion to be understood as an obvious inference.

Such abbreviated expressions of syllogism are called *Enthymemes*, and they are said to be of the first, second, or third order, according as major premise, minor premise, or conclusion is suppressed.

It is evident that no new logical principle is introduced. The whole syllogism is implied, though only part of it is put into words.

(ii) Sorites.—When several enthymemes follow each other in one chain of reasoning, the whole argument is known as a *Sorites* (a heap)—

$$\begin{array}{l} \textit{Every } S \textit{ is } X \\ \textit{Every } X \textit{ is } Y \\ \textit{Every } Y \textit{ is } Z \\ \textit{Every } Z \textit{ is } P \\ \hline \therefore \textit{ Every } S \textit{ is } P \end{array}$$

This is a chain in which the conclusion of every syllogism except the last is omitted. This suppressed proposition is the minor premise of the following syllogism. Thus, each of the intermediate syllogisms is represented by only one of its premises, and the bond of connexion is tacitly assumed throughout. If we analyse the chain, inserting these omitted propositions we get—

$$\begin{array}{lll} (1) & \textit{Every } X \textit{ is } Y & \text{(major)} \\ & \textit{Every } S \textit{ is } X & \text{(minor)} \\ \hline \therefore & [\textit{Every } S \textit{ is } Y] & \text{(concl.)} \end{array}$$

$$\begin{array}{lll} (2) & \textit{Every } Y \textit{ is } Z & \text{(major)} \\ & [\textit{Every } S \textit{ is } Y] & \text{(minor)} \\ \hline \therefore & [\textit{Every } S \textit{ is } Z] & \text{(concl.)} \end{array}$$

$$\begin{array}{lll} (3) & \textit{Every } Z \textit{ is } P & \text{(major)} \\ & [\textit{Every } S \textit{ is } Z] & \text{(minor)} \\ \hline \therefore & \textit{Every } S \textit{ is } P & \text{(concl.)} \end{array}$$

Such a sorites is named *Aristotelian*, as it was worked out by Aristotle.

Another form was elaborated by Goclenius, a sixteenth century logician, and is named after him the Goclenian. It reverses the order of the former, and in it the omitted conclusions are the major premises of the succeeding syllogisms.

Thus—

Goclenian Sorites.		*Analysis of Goclenian Sorites.*	
Every Z is P	(1)	*Every Z is P*	(major)
Every Y is Z		*Every Y is Z*	(minor)
Every X is Y	∴	[*Every Y is P*]	(concl.)
Every S is X	(2)	[*Every Y is P*]	(major)
∴ *Every S is P*		*Every X is Y*	(minor)
	∴	[*Every X is P*]	(concl.)
	(3)	[*Every X is P*]	(major)
		Every S is X	(minor)
	∴	*Every S is P*	(concl.)

In a given chain of reasoning it may be necessary to rearrange the order of the premises to get one of the recognised forms.

Either form of sorites may be entirely composed of hypothetical propositions. In that case—

(i) In the Goclenian Sorites the last premise may be categorical, and then the concluding enthymeme is the abridged form of a mixed syllogism, in which the categorical minor premise either affirms the antecedent, or denies the consequent, of the implied conclusion of the preceding syllogism; thus—

If **C**, *then* **D**,	*If* **C**, *then* **D**,
If **B**, *then* **C**,	*If* **B**, *then* **C**,
If **A**, *then* **B**,	*If* **A**, *then* **B**,
A,	**Not D**,
∴ **D**.	∴ **Not A**.

(ii) But in the Aristotelian Sorites the same result can only be obtained by adding to the sorites a categorical minor premise, and then regarding the implied conclusion of the preceding syllogism as the major, instead of the minor premise of the last syllogism. In other words, a mixed syllogism

at the end of a sorites must, in all cases, correspond to the Goclenian form; thus—

If **A**, *then* **B**,	*If* **A**, *then* **B**,
If **B**, *then* **C**,	*If* **B**, *then* **C**,
If **C**, *then* **D**,	*If* **C**, *then* **D**,
A,	**Not D**,
∴ **D**.	∴ **Not A**.

5. **Application.**

(i) Syllogistic Analysis.—As the relations and processes we have been examining are purely formal, we have deemed it wise to work them out with symbols, in which no accidental suggestion is present to obscure the issue. We will now give some illustrations of how reasonings may be analysed so as to set out explicitly the formal syllogistic relations.

As the examples show, the operative presence of syllogistic principles does not involve expression in strict syllogistic form. We can find that form, however, and this we could not do were it not already there implicitly. As the student of logic gains skill in the application of principles, formal analysis will become unnecessary, because the principles will increasingly direct his own thought, so that, whatever the words in which his arguments are expressed, they will become decreasingly likely to be inconclusive. At the same time, this tincturing of the thought makes it directly sensitive to the degree of cogency in the arguments of others. Automatic criticism is the aim, but it is the outcome of much deliberate criticism.

In setting out the syllogistic framework of an argument expressed with more or less redundancy of words; the relations, it may be, hinted at or suggested rather than clearly stated; some of the steps omitted; the whole, perhaps, wanting in sequence and proportion; we must strip away all

extraneous matter, restate in bald terms what was expressed with literary art, arrange the steps in sequence. Here are many opportunities for error. But one who has trained himself in turning the statements of colloquial speech into the skeleton-like normal propositions of formal logic will not find the difficulty insuperable.

When this reduction of ordinary speech to bare logical form is accomplished, each step of the argument should be examined in the light of the canons of syllogism. There is less danger of being led astray by accidental associations of terms if we carry the process of reduction to a skeleton form to its last stage, and express each term by a letter, as we do in solving problems in algebra.

Of course, not every piece of literature can be analysed syllogistically. Much is description or narrative, involving no inference. When inference is present, it is often inductive. Only deductive arguments can be analysed into syllogisms.

(ii) EXAMPLES.

(*a*) "A Jacobite, sir, believes in the divine right of kings. He that believes in the divine right of kings believes in a Divinity. A Jacobite believes in the divine right of bishops. He that believes in the divine right of bishops believes in the authority of the Christian Religion. Therefore, sir, a Jacobite is neither an Atheist nor a Deist." (*Dr. Johnson.*)

There are two distinct syllogisms. In each the minor premise is stated first. Each judgement is universal, for 'A Jacobite' means 'Any and every Jacobite,' and 'He that believes' means 'All who believe.'

Let J = Jacobite;
K = believer in divine right of Kings;
D = ,, a Divinity;
B = ,, divine right of Bishops;
C = ,, authority of Christian religion.

Then—

$$\begin{array}{ll} (1)\ \begin{array}{l} \textit{All } K \textit{ is } D \\ \underline{\textit{All } J \textit{ is } K} \\ \therefore \textit{All } J \textit{ is } D \end{array} & (2)\ \begin{array}{l} \textit{All } B \textit{ is } C \\ \underline{\textit{All } J \textit{ is } B} \\ \therefore \textit{All } J \textit{ is } C \end{array} \\ \text{(obverse)} \quad \textit{No } J \textit{ is not-}D & \text{(obv.)} \quad \textit{No } J \textit{ is not-}C \end{array}$$

Translating back—No Jacobite is either an Atheist or a Deist. The arguments are valid syllogisms in *Barbara*, in the first figure.

(*b*) "Now for the poet, he nothing affirmeth, and therefore never lieth; for, as I take it, to lie is to affirm that to be true which is false; but the poet never affirmeth; therefore, though he recount things not true, yet because he telleth them not for true, he lieth not."

Let P = poet; L = lying; A = affirming false to be true. Then—

$$\begin{array}{l} \textit{All } L \textit{ is } A \\ \underline{\textit{No } P \textit{ is } A} \\ \therefore \textit{No } P \textit{ is } L \end{array}$$

A valid syllogism in *Camestres*, in the second figure.

(*c*) "I believe all youths, of whatever rank, ought to learn some manual trade thoroughly; for it is quite wonderful how much a man's views of life are cleared by the attainment of the capacity of doing any one thing well with his hands and arms." (*Ruskin.*)

Stated baldly: All training of youths should include the acquirement of some manual skill, for such skill increases understanding of life [which is the aim of education].

Let Y = training of youths; S = acquirement of manual skill; L = clearing of understanding of life.

Then—

$$\begin{array}{l} \textit{All } S \textit{ is } L \\ \underline{\textit{All } Y \textit{ is } L} \\ \therefore \textit{All } Y \textit{ is } S \end{array}$$

The syllogism is formally invalid, for the middle term, L, is undistributed.

But, as with many syllogisms in the second figure with affirmative premises, it legitimately suggests the conclusion as probable. There may be other ways of cultivating a clear view of life; but, anyhow, the acquirement of manual skill is one. So we are first led to search for other ways, and then to compare them with this, in relation to the circumstances of each particular case.

(*d*) "If in order to be men we must be patriots, and patriotism cannot exist without national independence, we need no new or particular code of morals to justify us in placing and preserving our country in that relative situation which is most favourable to its independence." (*Coleridge.*)

Let M = men, including 'we'; P = patriot; N = favouring national independence.

Then—

$$\begin{array}{c} \textit{All } P \textit{ is } N \\ \underline{\textit{All } M \textit{ is } P} \\ \therefore \textit{All } M \textit{ is } N \end{array}$$

that is, we are justified from the premises given, without the need for a special code of morals "in placing . . . independence."

The syllogism is in the mood *Barbara.*

(*e*) "On the whole it seems to me certain that unless the area of Great Britain could be made larger than it is, or until the British people change their nature, a peasant proprietary is a dream. So long as a free energetic race of men are crowded together in a small space with every variety of employment open to them at home, with wide avenues to distinction offering themselves abroad, and with every individual striving to push his way to a higher station than that in which he was born, so long the ownership of land will be the luxury of the comparatively few." (*Froude.*)

Let C = countries in which "a free . . . born"; B = Great Britain; F = cases of few owners of land.

Then—

$$\begin{array}{r} \textit{Every } C \textit{ is } F \\ B \textit{ is } C \\ \hline \therefore B \textit{ is } F \end{array}$$

A valid syllogism in *Barbara*. An opponent must overthrow the major, or disprove the minor, premise.

(*f*) "Oppose Ministers, you oppose Government; disgrace Ministers, you disgrace Government; bring Ministers into contempt, you bring Government into contempt; and anarchy and civil war are the consequences." (*S. Smith.*)

Stated plainly: 'All contempt of constitutional government is conducive to anarchy; contempt of ministers is contempt of the executive government; therefore, contempt of ministers is conducive to anarchy."

Without expressing it symbolically it is evident that there are four terms, and the 'logical quadruped' offered is not a syllogism at all. The ambiguous use of 'Government' covers this in 'The Noodle's Oration,' that clever conglomeration of fallacies in which Sydney Smith parodied the political rhetoric current in his day, and by no means extinct in our own.

(*g*) "That the law may be a rule of action, it is necessary that it be known; it is necessary that it be permanent and stable. The law is the measure of civil right; but if the measure be changeable, the extent of the thing measured never can be settled." (*Johnson.*)

Let L = law; R = rule of action; K = known; P = permanent; M = measure of civil right.

There are two concurrent syllogisms—

$$\begin{array}{r} (1)\ \textit{Every } R \textit{ is } K \\ \therefore \textit{If any } L \textit{ is } R \\ \hline \textit{That } L \textit{ is } K \end{array} \qquad \begin{array}{r} (2)\ \textit{Every } M \textit{ is } P \\ \therefore \textit{If any } L \textit{ is } M \\ \hline \textit{That } L \textit{ is } P \end{array}$$

'All M is P' is the relation affirmed in "if the measure . . . settled."

Each syllogism is valid in the mood *Barbara.*

The whole structure illustrates the process through which the consequent is connected with its antecedent in a hypothetical proposition.

(h) "Happiness is not a mere permanent condition of mind. If it were, it would be within the reach of one who passed his days in sleep, living the life of a vegetable, or of one afflicted with grievous misfortune. As we cannot grant this, we must define happiness as some form of activity. But, as some activities are only desirable for the sake of something else, while others are desirable simply for their own sakes, it is clear that we must place happiness among those that are desirable in themselves, and not among those that are desirable for the sake of something else. For happiness lacks nothing; it is in itself all-sufficient." (*Aristotle.*)

Let H = happiness; C = permanent condition of mind;
R = within reach . . . misfortune; A = activity;
S = desirable for sake of something else;
D = desirable for itself.

Then—

$$\text{(1)}\quad \begin{array}{l} \textit{All } C \textit{ is } R \\ H \textit{ is not } R \\ \hline \therefore H \textit{ is not } C \end{array}$$

$$\text{(2)}\quad \begin{array}{l} H \textit{ is } A \textit{ or } C \\ H \textit{ is not } C \\ \hline \therefore H \textit{ is } A \end{array}$$

$$\text{(3)}\quad \begin{array}{l} \textit{All } A \textit{ is } S \textit{ or } D \\ H \textit{ is not } S \\ \hline \therefore H \textit{ is } D \end{array}$$

that is, Happiness is an activity desirable in itself.

The argument is valid. The first syllogism is **AEE** (*Camestres*) in the second figure; the second and third are mixed disjunctive syllogisms, in each of which the minor premise denies one of the alternatives in the major premise, and the syllogism legitimately concludes to the assertion of the other.

(*i*) "If obedience to the will of God be necessary to happiness, and knowledge of His will be necessary to obedience, I know not how he that withholds this knowledge, or delays it, can be said to love his neighbour as himself." (*Dr. Johnson.*)

Let O = obedience; K = knowledge; H = happiness; L = loving neighbour as himself; W = withholding.

Then—

$$\begin{array}{l} (1)\ \textit{If no } O\textit{, then no } H; \\ \quad \textit{If no } K\textit{, then no } O; \\ \hline \therefore\ \textit{If no } K\textit{, then no } H. \end{array}$$

$$\begin{array}{l} (2)\ \textit{If } WK\textit{, then } WH; \\ \quad [\textit{If } L\textit{, then not } WH;] \\ \hline \therefore\ \textit{If } L\textit{, then not } WK. \end{array}$$

Two valid pure hypothetical syllogisms, the first in mood **AAA** in the first figure, the second in mood **AEE** in the second figure. The major premise of (2) is implied by the conclusion to (1), and the minor premise of (2) is implied in the second part of the argument.

(*j*) "The common field-mouse destroys the nests of the humble-bee, so that if there are many field-mice the bees will be rare, and therefore the heartsease and clover will not flourish. But again, near the villages there are very few field-mice, and this is because the cats come out into the fields and eat them; so that where there are many cats there are few mice and many bees, and plenty of heartsease and Dutch clover. Where there are few cats, on the contrary

the mice flourish, the bees are destroyed, and the plants cease to bear seed and to multiply. And so you see that it actually depends upon the number of cats in the neighbourhood how many of these flowers there are growing in our fields." (*Buckley*, from *Darwin*.)

(1)	(2)
If few cats, many mice;	If many cats, few mice;
If many mice, few bees;	If few mice, many bees;
If few bees, little clover;	If many bees, much clover;
∴ If few cats, little clover.	∴ If many cats, much clover.

Each is a valid Aristotelian Sorites, composed of hypothetical propositions.

(*k*) "The chain of inference in this case is close and strong to a most unusual degree. A man desires that the actions of other men shall be instantly and accurately correspondent to his will. He desires that the actions of the greatest possible number shall be so. Terror is the grand instrument. Terror can work only through assurance that evil will follow any failure of conformity between the will and the actions willed. Every failure must therefore be punished. As there are no bounds to the mind's desire of its pleasure, there are, of course, no bounds to its desire of perfection in the instruments of that pleasure. There are, therefore, no bounds to its desire of exactness in the conformity between its will and the actions willed; and by consequence to the strength of that terror which is its procuring cause. Even the most minute failure must be visited with the heaviest infliction; and as failure in extreme exactness must frequently happen, the occasions of cruelty must be incessant.

We have thus . . . seen that the principle of human nature, upon which the necessity of government is founded, . . . leads on, by infallible sequence, where power over a community is attained, and nothing checks, . . . to that degree of cruelty which is necessary to keep in existence the most intense terrors." (*James Mill.*)

What is the line of thought? Nothing essential to it will be omitted if we write it in two syllogisms, thus—

(1) All men are desirous of power over others;
All rulers are men;

∴ All rulers are desirous of power over others.

(2) All desirous . . . others are terrorists to those subject to them;
All rulers are desirous . . . others;

∴ All rulers are terrorists to their subjects.

The syllogisms are formally valid, but they rest on an assumption as to the nature of man which is certainly not axiomatic, as Mill believed it to be.

In reviewing the book, Macaulay took other propositions given as axiomatic in it, and showed that by combining them an exactly opposite conclusion is inevitable. He wrote:—

"We will attempt to deduce a theory of politics in the mathematical form, in which Mr. Mill delights, from the premises with which he has himself furnished us. . . .

No rulers will do anything which may hurt the people.

This is the thesis to be maintained; and the following we humbly offer to Mr. Mill as its syllogistic demonstration:

No rulers will do that which produces pain to themselves;

but the unfavourable sentiments of the people will give pain to them;

therefore no rulers will do anything which may excite the unfavourable sentiments of the people.

But the unfavourable sentiments of the people are excited by everything which hurts them;

therefore no rulers will do anything which may hurt the people.

Which was the thing to be proved." [1]

These syllogisms are also valid.

[1] Review of Mill's *Essay on Government* in *Miscellaneous Works.*

6. Province of Syllogistic Reasoning.

We have chosen the last pair of illustrations to bring out the great danger of all deductive reasoning—the false assumption that the premises are both true and complete. By confining our premises to what supports the conclusion we wish to establish, we beg the question in an argument which, taken apart by itself, may be valid. As Macaulay says in this same review: "There is no proposition so monstrously untrue in morals or politics that we will not undertake to prove it, by something which shall sound like a logical demonstration from admitted principles."

Macaulay's own way of meeting Mill was to show that one part of the doctrine was inconsistent with another. This is the final test of all deductive argument—whether it lead to conclusions in harmony with the rest of knowledge. In short, deduction is an instrument for elaborating systems of thought. To take each little piece of reasoning apart and by itself, out of relation with everything in the universe and with all we know and think, is to commit in the very worst form the fallacy of wrongly estimating evidence.[1]

Nothing has brought so much suspicion upon logic as the habit of many logicians of ignoring the relations of arguments to matter outside themselves. Some, indeed, have even affirmed that logic has no concern with truth. 'If so,' says the plain man, 'logic has no concern for me, for I desire to know the truth.' The remedy, as has already been said, is not to throw overboard what is true in the investigation of the abstract and formal relations of thought, but to recognise its limitations. No argument which transgresses a syllogistic canon is valid; but many an argument which rigidly obeys those rules, and is formally impeccable, is a piece of false reasoning nevertheless, because it begins by begging the question as to what are its full and true grounds. Then it is worthless from the start.

[1] See pp. 132-133.

CHAPTER XV.

COMBINATION OF RELATIONS.

1. Nature of Inference Involved.

Of the eight main classes of relations enumerated at the beginning of the ninth chapter, those of Likeness and Unlikeness, Class Inclusion and Exclusion, Subject and Attribute, Causation, have been dealt with in the chapters on induction and deduction.

The inferences from the remaining types of relation cannot be analysed into syllogisms. In them there is no need to bring a particular case under a general law by finding an intermediary middle term; for the universal is immediately present in each. They have, therefore, simply to be brought together, and the consequences of the combination are apparent.

Of course, the same kind of relation must be involved throughout. It used to be—and, perhaps, still is—a favourite 'catch' among school-boys for one to ask another, with all seriousness, 'Do you say seven and five *are* eleven, or seven and five *is* eleven?' The emphasis put on the verbs was a sophistical fallacy.[1] Misled by it, the respondent frequently fell into the fallacy of irrelevance.[2] Taking the question to be grammatical and not arithmetical, he would answer, it may be, 'Oh, I say seven and five *are* eleven.' To which it would be replied, 'Well, *I* don't.' In contemptuous surprise

[1] Cf. pp. 107-108. [2] See pp. 109-110.

would come the query, 'What! do you say "Seven and five *is* eleven"?' To this, with a smile of superiority, the interlocutor would respond, 'Oh no! *I* say, seven and five are *twelve*'; thus himself committing a similar fallacy of irrelevance in respect to the intended point of the question as he had induced his friend to fall into. The reply of that friend was apt to be in another form of irrelevance; for 'assault and battery' may prove many things, such as anger, exuberant spirits, or superior strength of body; but can be no evidence of strength of reasoning. Grammar and arithmetic do not form one system; hence the confusion.

But when the relations are all of the same kind, their correct combination constructs the appropriate system, from which valid inferences may be directly drawn.

The special method of reasoning suitable to any set of relations is a question for the science concerned. Logic cannot include all human knowledge, and there is no knowledge apart from recognition of relations. The sole office of logic is to set forth conditions of validity. The combination of relations into complex systems, and the determination when, in any process, a principle has been violated, belong to the science which deals with that particular matter.

We will now illustrate by examples inference from combination of relations.

2. **Relations of Affinity.**

'If John is the brother of Mary, and Mary's son is named Henry, then Henry is John's nephew, and John is Henry's uncle.'

We reach the conclusion through a relation which holds universally—that sisters' sons are nephews, or, stated from the other end, that mothers' brothers are uncles. Immediately the system is constructed this relation is seen to be present in the particular case.

There is, then, no syllogistic process of bringing this case

under that universal relation. No intermediary middle term is implicit in the thought; consequently, none can be evolved out of it.

We are not usually aware of family relations removed more than a very few degrees. Yet such relations ramify into complex genealogical trees, and soon reach affinities for which we have no special names, because we do not need to refer to them in ordinary life. Their pertinence is seen when descent of property or title, or the renewal of an ancient barony, is at stake. The construction of the system, and the drawing from it the right conclusion as to hereditary right may then occupy heralds, lawyers, and judges, for many months or years.

In botany and zoology the following out of relations of affinity between classes is far from complete, but its mere adoption as the basis of systematisation has revolutionised those sciences. In them, 'affinity' is now taken in its literal meaning as implying common descent, and not, as it formerly was, in a metaphorical sense, as indicating nothing but closeness of resemblance, without any suggestion as to how such resemblance originated.

3. Relations of Quantity.

(i) Relations of Degree.—'A is greater than B, B is greater than C; therefore A is greater than C.' 'A is equal to B, B is equal to C; therefore A is equal to C.' 'John is more intelligent than Henry, who is more intelligent than William; so John is a good deal more intelligent than Henry.' 'My pianola-piano cost more than your American organ, and that cost more than your uncle's gramophone, so I am sure that the gramophone cost less than the piano.' 'The record of the thermometer shows that last night was colder than the preceding night, and that was colder than the night before; so the temperature of last night was a good deal lower than that of two nights back.' "This is not the case

of a possessor who has been, during many years, receiving great emoluments from land to which he had not a good title. It is the case of a possessor who has, from resources which were undoubtedly his own, expended on the land much more than it was originally worth. Even in the former case, it has been the policy of all wise lawgivers to fix a time of limitation [for questioning the title]. *A fortiori*, therefore, there ought to be a time of limitation in the latter case."[1]

The argument proceeds from one degree to another through an intermediate degree, and is known as the argument *a fortiori*. It is not syllogistic, for the intermediate degree is not a middle term. In the first example given, which is quite representative, the terms are '*A*,' '*greater-than-B*,' '*B*,' '*greater-than-C*.' But '*B*' is the name of a thing, while '*greater-than-B*' is the name of a relation. Things and relations cannot be identified with each other.

Many logicians, holding that all conclusive arguments are at bottom syllogistic, have attempted to reduce these to that form. All the attempts break down, for they are all beside the mark. The conclusion follows *immediately* from consideration of the combination of the relations. It is not deduced from a universal statement, such as 'What is greater than a greater is greater than a less.' That is nothing but the given premises generalised. Therefore, to use it in any form as a major premise is to beg the question.

(ii) Numerical Relations.—Relations of number may be stated indefinitely or definitely. We use such signs of number as 'a great many,' 'many,' 'most,' 'few,' 'very few,' 'hardly any,' 'the majority of,' 'a small minority of,' and so on, as well as such definite ones as 'a half,' 'two-thirds,' etc.

All these are reduced in formal logic to the indefinite 'some,' meaning 'some at least, it may be all.' This is adequate for the combination of propositions into syllogisms

[1] Macaulay: *Speech on Dissenters' Chapels Bill.*

of which the purpose is to bring cases under general laws, in order to reach conclusions as to whether or no those laws are established as universal. Syllogism has no concern with particular cases *as such*, that is, out of relation to a universal.

But in ordinary life we are interested in them. From a combination of two formally particular propositions '*Some S's are M*,' '*Some M's are P*,' no conclusion can be drawn: the union of such propositions does not make a syllogism, just because it cannot fulfil the purpose of syllogism. But, from combinations of signs of number which together exceed the total denotation of the middle term we can draw conclusions as to the particular cases in question. From 'Most of the congregation were late' and 'Most of the congregation consisted of women' we at least know that on that particular occasion some women were late.

If we are told that 'Half the congregation was late' and that 'Four-fifths of it was composed of women,' we can reach the more definite conclusion that, even in the extreme case of all the men present being late, at least three-eighths of the women who went to church at that time were unpunctual. If we learn that in a Zeppelin raid fifty people were killed, of whom forty per cent. were under ten, we can calculate that thirty persons above that age perished.

Such conclusions apply to the case in hand, but give no wider-reaching result.

We can also make inferences from order of succession. If James I. was the first Stuart king and James II. the fourth, it follows that there were two kings of that family between James I. and James II.

At bottom, all arithmetical calculations rest upon counting. And counting is the synthesis of equivalent units. We may use concrete things as counters, but counting in itself is merely the act of discrimination of one from another. Thus, its results hold universally. The sum of eight and four is

twelve, always and in all circumstances. The nature of the things counted do not come into the synthesis, which may, indeed, be simply of mental notings of succession.

The result of counting is to form a whole which is not simply the result of that synthesis, because it can be composed in other ways. Thus, $8 + 4 = 12$. But 12 is also the result of $7 + 5, 9 + 3, 20 - 8, 6 \times 2, 60 \div 5$, and of many other combinations of numbers. Further, 12 means a group of a particular size, an object of thought which has its own qualities, and can enter, in ways determined by them, into other combinations.

It follows that arithmetical calculations are abstract. They are accurate when the true relations between the numbers are kept. If in working a sum one made $7 + 5 = 11$, or gave the product of 9 and 3 as 24, the answer would be wrong. Such an arithmetical inference is invalid, because in the particular case the universal relation was not rightly apprehended.

The methods of combining numbers are the subject-matter of arithmetic. But, like all processes of inference, those methods are logical in the most general form of combining relations in a synthesis appropriate to the problem which is being solved.

4. **Relations of Space.**

We know all objects as existing outside us in space. When we abstract attention from everything about them except their spatial relations to each other, we can combine those relations, and so construct a spatial system from which we can draw inferences.

We know that the moon revolves round the earth, and the earth round the sun; hence, we can infer that the moon revolves round the sun. That 'The canopy is fixed over the throne, which is placed on a daïs, and the king sits on the throne' implies that he is upon the daïs, and under the

canopy. If B is placed between A and C, it follows that A is further from C than from B.

If we are told that Brighton is fifty miles south of London, and Portsmouth fifty miles west of Brighton, we can infer that Portsmouth is $50\sqrt{2}$ miles south-west of London. The relative positions of the towns can be represented by a right-angled isosceles triangle, and by trigonometrical calculation we find that the value of the hypotenuse of such a triangle is $\sqrt{2}$ times the length of the equal sides. Or we can reach approximately the same result by measuring its length either in the figure we construct or on a map. Of course, position involves both direction and distance. If either be omitted, no points are fixed, and so no definite system can be constructed.

The relation is always found exemplified in a particular case. It appears immediately the construction is made, for it is involved in the nature of the figure. So again, no analysis into syllogism is involved.

It is the same with all geometrical inferences. When we prove that the angles at the base of an isosceles triangle are equal to each other, we may draw a figure, mark its three angles by letters, and refer to it in every step of our reasoning. But the reasoning is concerned *only* with the constant relations which the figure exemplifies, not at all with the size, position, colour, or other accidental qualities, of the figure we draw.

This is the case in all mathematical reasoning. We attend exclusively to the relations, and these are the same for all corresponding cases. Hence, the relations we establish by our reasoning are constant or universal.

5. Relations of Time.

Events happen in time. They have, therefore, relations of simultaneity or co-existence, and of sequence or before and after.

If Wellington defeated Napoleon at the battle of Waterloo,

he must have been contemporary with him. If Wellington and Caesar did not live at the same time, neither can have defeated the other.

On such grounds Bentley rejected the *Epistles of Phalaris* as forgeries. He showed that " if the Epistles so belauded were true, Phalaris had borrowed money from men who lived three hundred years after his death, had destroyed towns that were not founded, and conquered nations that had no names."[1]

As examples of inference from sequence, we may take: 'The conquest of Constantinople by the Turks in 1453 was followed by a considerable migration of Greek scholars to Italy. They carried many manuscripts with them, and lectured on Greek literature in Italian towns. So, a great revival of the study of Greek in Italy was a consequence of the fall of Constantinople.' 'Slavery leads to wasteful cultivation, and this leads to economic distress, therefore, slavery is economically unwise.' 'War involves excessive expenditure. To meet this, heavy taxation must be imposed and great loans raised; to pay the interest on these loans the taxation has to remain heavy long after peace has been made. Therefore, war leads to long-continued heavy taxation.' 'The Black Death caused a great dearth of labour. This induced a considerable and general rise of wages. Therefore, the Black Death led to a general rise of wages.'

Here, too, it is seen that the syllogism is not an analysis of the process of thought involved. There is no reference of particular cases to general laws through the intervention of a middle term. The conclusion is derived immediately from the union of the given relations in a time-system.

[1] Craik: *Life of Jonathan Swift*, vol. i., p. 88.

CHAPTER XVI.

EXPLANATION.

1. Importance of Understanding.

We observe, inquire, think, in order that we may understand. For, we can make no use of anything unless we understand it, in the way, and to the extent, necessary to our purpose. Nor are our interests and purposes always practical. We all have the instinct of curiosity—or prompting to find out—and that leads us to seek knowledge for its own sake, that is, without intention to turn it to some practical use.

True, this may be mainly a wasted force. Many may exercise their curiosity only on the trivial: try to peer into their neighbour's private concerns, but never to pierce the veil that covers the mysteries of nature and of life. "The desire for logical consistency is not one which presses forcibly upon the less cultivated intellects. They do not feel the necessity of unifying knowledge or bringing their various opinions into consistency and into harmony with facts." [1]

One result of a serious study of the processes of thought should be to direct it to problems really worth solution. Surely, it is of the greatest moment to a man whether he see into the meaning of life, even if dimly, or whether he never

[1] Sir L. Stephen: *The English Utilitarians*, vol. i., pp. 6-7

get beneath the surface; whether he set himself to learn what is good and what is evil, what is true and what is false, what is of worth to men and what is worthless, or whether he fritter away time and energy in seeking to know things of little or no importance at the moment, and doomed to pass into oblivion to-morrow; whether he feed and exercise his mind, or whether he surfeit it with sweets, and allow it to lounge through life, never putting forth effort, and so becoming ever flabbier, and ending with 'nothing attempted, nothing done.'

2. **Nature of Explanation.**

(i) Explanation and System.—We understand when we reach a reason or explanation. If I find a window broken, and a stone lying on the floor, the breaking of the window is explained, though the throwing of the stone may be unexplained. Explanation would trace that to some particular thrower, and would make plain why he threw it. So long as the stone-throwing is simply a fact by itself it is unexplained. When it is seen as one in a system of related facts, we understand it.

So always, explanation is reference to a system.

But the system must be an appropriate one, or the reference is sham, not real. If Tommy come home with a large bump on his forehead, he would give an adequate explanation if he said he had been fighting with Johnny, or that he had run against a lamp-post in the dark. But if he alleged that his intellectual exertions at school had been so strenuous that they had caused his head to swell, his explanation would not be accepted. Yet he would have referred the fact to a system in this as in the other cases. The difference is that those systems were in agreement with our experience, but that this is quite unfamiliar.

(ii) Analogy in Explanation.—To each one of us the explanation of a new fact means its reference to a familiar system. The reference may be direct, as in the cases cited.

Or it may be indirect, through an analogy or simile. This tells us that a relation already familiar to us, as holding between known facts, also holds between the new facts of which we seek explanation.

We all know how much light an illustration often throws on an obscure point. As old Thomas Fuller put it, "Indeed, reasons are the pillars of the fabric of a sermon: but similitudes are the windows which give the best lights."[1]

Bolingbroke's simile: "The members of the House of Commons grow, like hounds, fond of the man who shows them game, and by whose halloo they are to be encouraged," throws a bright light on the popularity of certain politicians, otherwise inexplicable, and helps us to see why gentlemen, kindly and fair in private life, should be prone to unfair insinuation and vituperation in politics. It makes their conduct an exhibition of the hunting instinct, not extinct in man, though usually dormant in reference to his fellows.

Analogy, then, helps us to understand. At times we misunderstand. Misled by some accidental features in the analogy, we refer the fact to a wrong system, or to the right system in a wrong relation.

(iii) Explanation and Knowledge.—All this is personal. Explanation enables some person to understand. And what anyone can understand depends on what he knows, for explanation is placing in a familiar system.

We all know that what is explanation to one is unintelligible to another. An explanation of some mathematical problem that would be fully satisfactory to a few highly trained mathematicians might be wholly incomprehensible to the vast majority of mankind. A minority would be able to see something of its bearing; a smaller minority would, perhaps, get enough light to guide their own efforts till they reached the explanation themselves.

[1] *Holy State*, bk. ii., ch. ix., 11.

In the contrary case, explanations which satisfy the ignorant do not satisfy the learned. That the leaves fall from the trees because winter is coming on is a sufficient explanation to a child, and to many an adult. It is a familiar sequence, having its place in a time-system. But a botanist would say it is no explanation at all.

Similarly, the explanations of one age become inadequate, or even absolutely false, to men of a subsequent time, when knowledge of that kind of event has increased. For many centuries the explanation that the happenings in a man's life are determined by the influence of the planets was generally received. Now it is rejected by most people, at any rate in what we call civilised lands, though it seems still to find acceptance with a few. Why this general change of view? Because of increase of astronomical knowledge. But is that knowledge in your mind, or in mine? Few of us could answer 'Yes.' We reject the claims of astrology because we are told by those whom we believe best able to judge that they are unfounded.

Equally do we accept many explanations on the like authority. We could not reach them ourselves, because we have not enough pertinent knowledge.

When we do reach an explanation by our own efforts, we feel that we understand it in a more living way, that we can apply it more readily and surely to the solution of other problems than when we receive it from another.

In the latter case, indeed, we have no explanation unless we can refer it to knowledge which is really our own. It does not matter how we obtained that knowledge. It may be the outcome of our own investigations, or it may have been received from others. The important question about knowledge is not its origin, but its fullness and accuracy.

In every case, then, explanation is the bringing of new facts into relation with existing knowledge.

3. **Popular and Scientific Explanation.**

(i) Scope.—We are always seeking explanations in daily life. Often the matter is of but momentary importance. Then we do not push the investigation very far. My thirst for explanation of my broken window is slacked when I discover the boy who threw the stone, and find that it was a case of bad aiming at a companion. I do not ask why he wished to hurt his fellow. Still less do I think it needful to enter on a study of boy-nature in general, or of the mental and moral history of this boy in particular. Nor do I inquire into the qualities of glass and of stones, the laws of impact, momentum, and trajectory. Yet all these are pertinent to the question.

Practically, then, we limit our search for explanation by the needs of the moment. The more important the case, the further we push them. And we measure importance rather by reference to the future than by consideration of the past. If the explanation of a present event is likely to be a valuable guide to future conduct, we desire to understand the conditions under which it occurs more fully than if its consequences seem to be exhausted.

It is evident, then, that explanation may be pushed to very various lengths. Here comes in the difference between the explanations we seek of the ordinary events of daily life, and the explanations sought in science. The aim of science is to increase accurate knowledge as much as possible. It, therefore, sets no limit to the search for explanation. Nor does it fail in patience in seeking it. Scientific explanation is always extending. Newton's theory of gravitation explained the motions of the moon and planets as cases of the operation of the same law that accounts for the fall of a stone. Now it is established that the same relation holds between the smallest particles of matter, and that it is a reciprocal attraction.

(ii) Classification and Explanation.—Classification is a step towards explanation, because it systematises knowledge of things according to their resemblances. When it takes as its basis a formative principle, such as evolution, or chemical combination, it advances yet further. In chemistry classification passes beneath the surface. Not the objects of daily observation, but the elements of which they are composed, are the ultimate things it seeks, and it groups the common objects of experience according to the combinations of those elements which constitute them. Biology seeks to push its classification to the simplest forms of organised life, and to make the modifications of those forms the grounds of the grouping of existing organic beings.

(iii) Law and Explanation.—Merely to classify, however, is not to give explanation. The great question for the biologist is the determination of the operative forces which induced the evolution of present forms of organised beings from the simpler forms of the past. The chemist seeks to know, not only what elements are combined in an object we can see or handle or taste or smell, but in what proportions they are combined, under what conditions the combination takes place, remains stable, or is liable to decomposition. The answering of such questions is explanation.

Events which present themselves for explanation are referred to familiar causes. But in such reference, the idea of the nature of that kind of causal relation is always enlarged and made more precise. There is a general drawing together of events, on the surface diverse, as combinations of the same far-reaching laws. "In the remotest double star which the telescope can divide for us, we see working the same familiar forces which govern the revolutions of the planets of our own system. The spectrum analysis finds the vapours and the metals of earth in the aurora and in the nucleus of a comet."[1]

[1] Froude: *Scientific Method applied to History*, in *Short Studies*, vol. ii.

Advance in explanation, then, is the finding that the constant relations with which we have become familiar have a wider scope than we had hitherto attributed to them. This advance is progressive. Hence, we can think of the universe only as a vast system, combining an infinite number of systems of all imaginable grades and scopes.

Our various explanations stop in one or other of those systems, but as we press forward we advance to wider and wider systems, in which those we at first took to be self-contained and ultimate in their own line are seen to be but elements. Chemistry and physics merge into each other in physical chemistry, and the outlook is that such a process may be continued indefinitely. The ultimate goal would be the whole universe conceived as a single, all-embracing system. That is far from being in sight, but it marks the stage at which explanation would be complete.

(iv) Verification and Explanation.—The danger here is a premature fusion of different branches of knowledge. It is possible that in time the boundary that marks off the inorganic from the organic systems of the world may be passed. At present, no way of doing this has been found, and it is illegitimate to assume that this boundary is not real before it has been definitely shown to be only apparent.

So with the division between the mental and the physical. The laws discovered in the workings of mind are quite different from those of physics. Whether in the future each may be shown to be a manifestation of the same more fundamental laws we cannot say. We do not know that there are such underlying laws. We do not even discern the line of approximation. But till we do, it is illegitimate to assume that the laws of mind and of matter must be at bottom identical, on the strength of a theory that the ultimate laws of all existence must be few and simple.

Still more illegitimate is it to assume that the laws of physics *are* laws of mind. Yet that has been done. Men

have believed that mind is but a by-product of brain, and mental life but a reflexion of the transmutations of physical energy.

Valid explanation is cautious. It carefully confines itself to what can be shown to be true. We do not explain, but dream, when we allow fancy to take far-reaching flights, of which the only justification is their attractiveness.

(v) Imagination and Explanation.—Yet imagination has a part to play in explanation. Fancy is imagination without a foundation. True imagination is the effort of the mind to find explanation. It supposes facts to be related in certain ways, and then asks experience whether the supposition is justified. All supposition, or hypothesis, is imaginative construction. Imagination passes beyond the facts, but it does not throw them on one side. When the products of imagination are found to represent real things and occurrences, the truth of the imaginative flight is established.

(vi) Goal of Explanation.—Thus we see that all induction is search for explanation. Deduction is the application of a proved explanation to fresh cases. For explanation is the discernment of the constituent relations which bind fact to fact in system. Beyond this it cannot go.

It is profitless to ask why elements and relations are what they are. Why do material bodies attract each other? We refer to gravitation, which expresses the constitution of the material system of the universe in that respect. That is ultimate for science, and must be accepted by it as the basis of all that needs explanation. To seek to go further is to ask for an explanation of the universe outside the universe. But for experience the universe is all. There is no passing beyond it. It must be accepted as the ultimate fact, and as the giver of all meaning. Scientific explanation tries to answer the question: The universe being what it is, why does such and such an event take place in such and such a way?

4. Value of Thought.

The one work of thought is, then, to enable us to understand our experience, and, through such understanding, to profit by it. To think clearly is to see things and events as they are. This is no small gain. But this very clearness of vision shows nothing more plainly than that thought is not the only influence in life. We feel, desire, and will, as well as think. And we think because we desire and will. Thought is the instrument by which we plan to attain our ends. Thought helps us to criticise those ends.

But thought is not the ultimate arbiter of our fate. A man who was nothing but a thinking machine would be a monster, no matter how perfect a machine he might be. We love and hope, we value beauty as well as knowledge, we recognise inner promptings to good and evil, and it is not thought alone that decides between them. To learn to think clearly is good: to do nothing but think clearly would be bad, were it possible.

All this is shown by investigation into thought itself. For that shows the bounds and limitations, as well as the power of thought. Logic results from such an investigation. Its use is both positive and negative. Positive, in laying bare the conditions of clarity of thought; negative, in showing that much of what is of the highest value in life is beyond its province. It would be absurd to assert that logic is the one thing needful for the successful guidance of life: yet more absurd would it be to deny it the value we have tried to show that it possesses.

GLOSSARY.

Logic has been overladen with technical terms, many of which mark distinctions of little or no logical importance. As, however, they still find a place in examination papers, we append a brief explanation of the more common of them.

1. Propositions.

(i) Propositions are divided into two classes—

(*a*) those in which P is contained in the definition of S;

(*b*) those in which P is not so contained.

They are known by several pairs of names—

(*a*) analytic,	*a priori*,	explicative,	verbal.
(*b*) synthetic,	*a posteriori*,	ampliative,	real.

This distinction belongs to epistemology, or theory of knowledge.

(ii) Propositions are said to be—

(*a*) Of *First Intention* when they express a direct perception, as 'That is a dog.'

(*b*) Of *Second Intention* when they express a relation between things as 'The dog is the friend of man.'

This distinction is psychological.

[N.B. Intention (*intentio*) = an act of mind.]

(iii) Propositions are said to be—

(*a*) *Secundi adjacentis* when copula and P are not separated;

(*b*) *Tertii adjacentis* when copula and P are separated (*see* pp. 52-53).

(iv) Propositions are called *Indesignate*, or *Preindesignate*, when no sign of quantity is attached to S (*see* pp. 67-68, *l*, *m*, *n*, *o*).

(v) Propositions of the form S *is not-P* have been regarded by some logicians as forming a third species with Affirmative and Negative, and named *Limitative*, or *Infinite*. Affirmation of a negative term is, however, an indirect way of denying a positive term; the form of statement is a question of language, not of thought (*see* pp. 59-60).

(vi) COMPOUND PROPOSITIONS are—

(*a*) *Copulative*, a simple combination of affirmatives; '*and* . . .' (*see* pp. 64-65, *e*).

(*b*) *Remotive*, a simple combination of negatives; '*neither* . . . *nor* . . .' (*see* p. 65, *f*).

(*c*) *Discretive*, adversative combination of affirmatives; '*but*,' '*although*.'

The question is grammatical.

(vii) EXPONIBLE PROPOSITIONS express a complex idea in a simple form. They are said to be—

(*a*) *Exclusive*, marked by *alone*; 'The virtuous alone are happy.'

(*b*) *Exceptive*, marked by *except*, *but*, etc.; 'None but the virtuous are happy.'

The distinction between these is wholly one of language (*see* p. 65, *g*, *h*).

(viii) PERMUTATION, an alternative name for *obversion*.

2. **Terms.**

(i) INCOMPATIBILITY OF TERMS has three degrees—

(*a*) *Contrary*, or *Opposite*, extremes of difference; giant, dwarf; black, white; virtuous, vicious.

(*b*) *Contradictory*, simply negating each other; giant, non-giant; black, not-black; equal, unequal.

(*c*) *Repugnant*, mutually exclusive without being (*a*) or (*b*); black, blue, grey; woollen, silken.

A further distinction is sometimes made by speaking of terms as—

(*d*) *Privative*, when they imply the absence of an attribute in a subject capable of possessing it; blind, dumb.

(*e*) A formally negative term, *not-S*, if not referred to any universe of discourse, is called *Infinite* or *Indefinite*.

Any interest these distinctions may have is grammatical.

(ii) Terms denoting materials are sometimes called *Substantial*; gold.

(iii) *Fundamentum Relationis*—the relation implied by a pair of relative terms.

(iv) A distinction is also drawn between terms that have but one meaning and those that have more than one. The former are called *Univocal*, the latter *Equivocal*, when the same word happens to refer to a plurality of quite different things; as 'page,' a boy-servant, and a piece of paper. When the meanings partly overlap, the terms are said to be *Analogous* (*see* pp. 31-34).

This is altogether a question of language, and touches logic only so far as ambiguity of language causes confusion of thought. Logic demands that the different meanings be disconnected, and that each be recognised as a separate term.

(v) Words that can stand alone as terms are called *Categorematic*, those that cannot, *Syncategorematic*. This is wholly a matter of grammar.

3. **Fallacies.**

(i) Classification.

(*a*) Aristotle classified sophistical fallacies into—

Sophismata in dictione, arising out of ambiguity of language; those discussed in Chapter II.

Sophismata extra dictionem, those which can be detected only by examination of the matter—*Consequens* (*see* Chapter V., § 5 (iii) (*e*), and those in Chapter VII., §§ 2, 3 (i) (*a*, *b*, *c*, *d*), (ii), 4 (i), (ii).

(*b*) Whately divided into—

(1) *Logical*, "Where the conclusion does not follow from the premises," including—

(*α*) *Purely Logical*, which exhibit their fallaciousness by the bare form; all obvious violations of syllogistic rules; *see* Chapter XIII., 3 (ii) (*c*), (iv).

(*β*) *Semi-logical*, "all cases of ambiguous middle term except its non-distribution"; see Chapter XIII., 3 (ii) (*b*).

(2) *Material* or *Non-logical*,—*ignoratio elenchi* and *petitio principii*, the latter including as a sub-class *Non sequitur*.

(ii) Special Cases of Ignoratio elenchi:

(*a*) *Argumentum ad baculum*—appeal to force.

(*b*) *Argumentum ad ignorantiam*—appeal to ignorance.

(*c*) *Argumentum ad judicium*—appeal to accepted opinion.

(*d*) *Argumentum ad misericordiam*—appeal to pity.

(*e*) *Argumentum ad populum*—appeal to popular prejudice.

(*f*) *Argumentum ad verecundiam*—appeal to respected authority

(iii) *Hysteron proteron*—assuming conclusion as premise in a single syllogism (*see* p. 118).

(iv) *Non causa pro causa*, *Non per hoc*, *Non propter hoc*—other names for *Non sequitur* (*see* Chapter VII, (4) (i)).

[N.B. '*Causa*' = *causa cognoscendi* or reason, not *causa essendi* (*see* pp. 10-11).]

(v) The fallacy *a dicto simpliciter . . . quid* (*see* pp. 130-131) has been confused by some writers, *e.g.* De Morgan, with the fallacy of *Accident* (*see* pp. 129-130). Then the fallacy *a dicto sec. quid . . . simpliciter* is regarded as a *Converse Fallacy of Accident.*

4. Classification and Division.

(i) *Metaphysical Division*—another name for Conceptual Analysis (*see* p. 170).

(ii) *Classification and Definition by Type*—a kind of description which marks out a class by a typical example.

(iii) *Ignotum per ignotius*—a definition expressed in terms more obscure than that to be defined.

(iv) *Real Definition*—one which declares the nature of a thing.
Nominal Definition—one which declares the meaning of a word.

As all words are defined in relation to facts, this distinction has no logical significance.

5. Induction.

(i) *Consilience of Inductions*—when inductions from one class of facts supply an unexpected verification of another class of facts.

(ii) *Derivative Law*—A law inductively ascertained, which is explained by being shown to be deductively deducible from a wider law.

(iii) *Quantitative Induction*—an induction which determines the causal relation *quantitatively*, and gives corresponding values to cause and effect.

6. Syllogism.

(i) *Proximate Matter of a syllogism*—the three propositions which compose it.

(ii) *Remote Matter of a syllogism*—its terms.

(iii) When the conclusion of a syllogism is put forward as a thesis to be proved, it is called the *Question*, and the premises, then introduced by 'because,' the *Reason.*

(iv) General Principles:

(*a*) *Nota notae*—the general principle of the first figure expressed in terms of connotation: 'Whatever is a mark of any mark, is a mark of that of which this last is a mark.'

(*b*) Special statements of the fundamental principle of syllogism have been devised for each of the 'imperfect' figures, viz.—

Fig. ii: *Dictum de diverso*—'If one term is contained in, and another excluded from, a third term, they are mutually excluded.'

Fig. iii: *Dictum de exemplo*—'Two terms which contain a common part partly agree, or if one contains a part which the other does not, they partly differ.'

Fig. iv: *Dictum de reciproco*—'If a term be included in a second term which is excluded from a third, then the third is excluded from the first; if a term be included in, or excluded from, a second term which is included in a third, then a part of the third is included in, or excluded from, the first.'

These are more ingenious than profitable.

(v) Chains of Reasoning.

Prosyllogism—a syllogism whose conclusion is a premise in the syllogism with which it is connected.

Episyllogism—a syllogism one of whose premises is the conclusion of the syllogism with which it is connected.

Progressive Reasoning—a chain of reasoning in which we proceed from prosyllogism to episyllogism. All sorites are examples.

Regressive Reasoning—a chain of reasoning in which we proceed from episyllogism to prosyllogism.

Epicheirema—a regressive chain of reasoning abridged by the omission of one of the premises of each prosyllogism.

Abscissio infiniti—a gradual process of exclusion of the alternatives of a disjunctive proposition by mixed syllogisms, till only one is left, *e.g.*

$$\begin{array}{l} S \text{ is } P \text{ or } Q \text{ or } R, \\ S \text{ is not } R \\ \hline \therefore S \text{ is } P \text{ or } Q, \\ S \text{ is not } Q \\ \hline \therefore S \text{ is } P. \end{array}$$

(vi) Mixed Hypothetical Syllogisms.

Both the *Modus Ponens* and the *Modus Tollens* are subdivided into four classes, called in each case *Modus ponendo ponens*, when both minor premise and conclusion are positive; *M. tollendo tollens*, when they are both negative; *M. ponendo tollens*, when the former is positive and the latter negative; *M. tollendo ponens*, when the former is negative and the latter positive.

QUESTIONS AND EXERCISES.

[*Nearly three-fourths of the following questions and exercises are selected from elementary examination papers set at various universities.*]

CHAPTER I.

1. Distinguish between the strict and the loose use of the word 'thinking.' Explain the nature and value of thinking in the stricter sense.

2. Explain and examine: 'The form is not form that can dispense with every matter, but that is independent of this or that special matter.'

3. Comment on: 'Art and Science are not found separate. In no place is anything to be done, but in the same place there is something to be known; in no place is anything to be known, but in the same place there is something to be done.'

4. Give several instances of *causae cognoscendi* (*a*) which are, (*b*) which are not, *causae essendi.*

5. Annotate the following statements:—

(*a*) 'Logic is concerned, not with the matter of thinking, but only with its forms.'

(*b*) 'Once true, always true; once false, always false.'

(*c*) 'The law of identity is, that what is true in one context is true in another.'

(*d*) 'Reality is one and self-consistent (*identity* and *contradiction*); a system of reciprocally determinate parts (*excluded middle*); and a system of reciprocally determining parts (*sufficient reason*); every part or feature of reality may be regarded as a consequent to which some other part or parts, or ultimately the whole, stands as ground.

6. 'Logic is the *science* which investigates the general *principles* of *valid thought.*' Explain carefully each of the italicised words in this definition

7. 'Logic is to be distinguished from positive sciences on the one hand, and from practical arts on the other.' Explain this, and state what you consider to be the general character of Logic.

8. "Trust," said Lord Mansfield to Sir A. Campbell, "to your own good sense in forming your opinions; but beware of attempting to state the grounds of your judgements. The judgement will probably be right—the argument will infallibly be wrong."

What inference would you draw as to Sir A. Campbell's previous training? Explain the possibility of the advice being good.

9. Compare logic and dialectic, and distinguish between their aims.

10. Explain and illustrate: 'Logic has a unique character of its own, and is not a mere branch of Psychology, though psychological and logical discussions are no doubt apt to overlap one another at certain points.'

CHAPTER II.

1. Show how the process by which we acquire language leads to frequent ambiguities of meaning.

2. Though a botanist's knowledge of plants differs from that of a gardener, they can converse about plants. How is this?

3. Does logic deal with language or with thought?

4. What special difficulties has research in history to encounter?

5. "In another isle there are dwarfs, which have no mouth, but instead of their mouth they have a little round hole, and when they shall eat or drink, they take it through a pipe, or a pen, or such a thing, and suck it in." (*Mandeville.*)

Test the value of this testimony.

6. Estimate the force of the following: "Events which, if they ever happened, happened in ages and nations so remote that the particulars could never have been known to him, are related [by Hero-

dotus] with the greatest minuteness of detail. . . . We are, therefore, unable to judge whether, in the account which he gives of transactions respecting which he might possibly have been well informed, we can trust to anything beyond the naked outline. . . . The great events are, no doubt, faithfully related. So, probably, are many of the slighter circumstances ; but which of them it is impossible to ascertain. The fictions are so much like the facts, and the facts so much like the fictions, that, with respect to many most interesting particulars, our belief is neither given nor withheld, but remains in an uneasy and interminable state of abeyance. We know that there is truth ; but we cannot exactly decide where it lies." (*Macaulay.*)

7. Estimate the value of the following evidence : "A short engagement took place, during which one of the enemy torpedo-boat destroyers was sunk." (*Official Report*, Jan. 24, 1917.) "How many British ships were in the immediate neighbourhood it is impossible to say. Some of the wounded [sailors of a damaged German torpedo-boat taking refuge at Ymuiden] say they saw 12, some 60, and some 100. . . . The captain had both legs cut off, and was carried below, where he died in a few minutes. . . . The wounded say that seven of their boats were certainly sunk. . . . *Later.*—The commander was not killed, as at first reported." (Correspondent of *The Daily Chronicle*, Jan. 24, 1917.)

8. Comment on : 'If ten credible witnesses agree in their testimony to a fact, the value of their concurrent testimony is more than ten times the value of the testimony of each.'

9. Point out the fallacies of ambiguity in :—

(*a*) 'The events narrated by Homer are credible : therefore Homer is a credible witness.'

(*b*) "All criminal actions ought to be punished by law ; prosecutions for theft are criminal actions ; therefore prosecutions for theft ought to be punished by law." (*De Morgan.*)

10. 'I will do it *presently*.'

'We are sore *let* and hindered.'

'*Prevent* us, O Lord, in all our doings.'

What misunderstandings might arise from confounding the present with the Elizabethan meanings of the words in italics?

11. Distinguish by actual instances between the collective and distributive use of terms. What fallacies are due to a neglect of this distinction?

12. Distinguish and correct ambiguities in :—

(*a*) "Every corridor in the Hotel answers to Euclid's definition of a straight line." (Advt. in *The Field*, 1910.)

(*b*) "The large comet was seen by a resident in the heavens in the direction of the Forest." (*The Football Echo*, 1910.)

(*c*) "There was little Ernest Hunter, whose indescribable hat covered a head that must have knocked around the world considerably before he found it." (Mr. V. Grayson in *The Clarion*, Feb. 1910.)

(*d*) "In the next compartment was the wife of a prominent politician, off to the Riviera. Her husband, seeing her off, looked wistfully after the train as it pulled slowly out of the station with its heavy load." (*The Daily Mail*, Jan. 1910.)

13. "*Thou* shalt not bear *false witness against thy neighbour*." Distinguish the six meanings obtained by emphasising in turn each of the italicised words.

CHAPTER III.

1. Explain :—

(*a*) 'We do not always talk in propositions.'

(*b*) 'Though a man may deliberately deceive others, he never willingly deceives himself.'

(*c*) 'All denial rests on an implicit affirmation.'

(*d*) 'All judgement at once analyses and synthetises.'

2. Explain and illustrate 'Universe of Discourse.'

3. Why is it sometimes necessary to take account of context in interpreting propositions? Illustrate your answer.

4. What is the logical import of *all*, *some*, *if* . . *then*, *either* . . . *or*?

5. Explain and illustrate: 'Terms may often be found which go together in pairs, in the sense of their being mutually exclusive and collectively exhaustive in their application.'

6. How are the quantity and the quality of propositions indicated in logical analysis? Transform the following propositions so as to bring out their logical nature :—

(*a*) 'Honesty is compatible with ignorance.'
(*b*) 'Selfishness may exist without prudence.'
(*c*) 'Vice never brings happiness.'
(*d*) 'Lying is a sure sign of guilt.'

7. Explain and illustrate the distinctions between Categorical, Hypothetical, and Disjunctive, Propositions.

Show how (*a*) a Hypothetical Proposition can be expressed as a Categorical, (*b*) a Disjunctive Proposition can be expressed as a Hypothetical.

8. Explain and illustrate what is meant by a Generic Judgement.

9. Explain and exemplify the *Modality* of Propositions.

CHAPTER IV.

1. Distinguish (*a*) between a *word*, a *name*, and a *term*; also (*b*) between a *sentence* and a *proposition*. Can a term or a proposition be illogical? Illustrate your answer.

2. Distinguish carefully between the *denotation* and the *connotation* of names, and explain how far it is possible to know one without the other.

3. Explain and criticise the following statement: 'As the intension of a term is increased the extension is decreased.'

4. Give, in terms of Denotation and Connotation, two different views as to the precise import of the predication in the proposition 'Some plants are evergreens'; stating, with your reasons, which of these views you prefer.

5. What do you understand by the connotation of a term? In what way does the connotation of *means of locomotion* differ from that of *aeroplane*, and the connotation of *virtuous* from that of *temperate*?

6. Consider whether the following names are connotative: (*a*) next month, (*b*) a mile, (*c*) the Great Bear.

Are any Abstract Names connotative?

7. Distinguish between (*a*) singular and general terms, (*b*) concrete and abstract terms, giving two examples of each. To which of the latter classes are *adjectives* assigned and why?

8. Discuss the distinctions (*a*) between Collective and Distributive, (*b*) between Positive and Negative, Terms. Illustrate your discussion by examining the following statements:—

(*a*) 'All the plays of Shakespeare cannot be read in a day.'
(*b*) 'Anyone who is not industrious must be accounted idle.'

9. What is a Concrete Name? Consider whether the following names are concrete: circle, selfish, relation, susceptibility. Can every concrete name be the subject of a proposition?

10. State, giving reasons for your opinion, whether the following terms are (*a*) positive or negative, (*b*) connotative or non-connotative: Loch Katrine, uncommonly good, the syllogism, irreverence.

11. Give examples (*a*) of a pair of terms that are opposite to one another, (*b*) of a pair of terms that are correlative to one another. What kinds of terms have opposites and what kinds of terms have correlatives?

12. What are the logical characteristics of the following terms: House of Commons, Adam Smith, reasoning, the discoverer of the course of the Congo?

13. State carefully the logical peculiarities of the following italicised words:—

(*a*) '*That* is excellent.'
(*b*) '*Nothing* is a trouble to him.'
(*c*) 'His play was *uninteresting*.'
(*d*) '*Cleanliness* is indeed next to godliness.'

14. Indicate by the technical symbols the quantity and the quality of these propositions:—

(*a*) 'Gentle words are always gain.'
(*b*) 'He was not all unhappy.'

(*c*) 'Never yet was noble man but made ignoble talk.'

(*d*) 'A bard may chant too often and too long.'

(*e*) 'In England all things not forbidden are permitted; abroad all things not permitted are forbidden.'

(*f*) 'There is a reputation talent alone can win, and which yet is not always the reputation of talent.'

(*g*) 'The four quarters, South, East, West, or North, All are alike to me.'

(*h*) 'I alone have found the truth.'

15. Determine the quantity and the quality and give the logical form of these propositions :—

(*a*) 'All men have not faith.'

(*b*) 'What a woman thinks of women is the test of her nature.'

(*c*) 'Relatives are ready-made friends.'

(*d*) 'What heart-breaking torments from jealousy flow,
Ah! none but the jealous—the jealous can know.'

(*e*) 'All that we know is, nothing can be known.'

(*f*) 'The agreement of deduction with induction yields the strongest proof.'

(*g*) 'We'll all keep a corner.'

(*h*) 'The tree is my seat that once lent me a shade.'

(*i*) 'Most rodents are herbivorous.'

(*j*) 'Either thou art most ignorant by age, or thou wert born a fool.'

16. Determine the quantity and the quality and give the logical form of these propositions :—

(*a*) 'All's to be feared where all is to be gained.'

(*b*) 'The reward of one duty is the power to fulfil another.'

(*c*) 'Man only doth not join with nature in her homage.'

(*d*) 'No enmity is like domestic hatred.'

(*e*) 'Virtue is but selfishness behind a mask.'

(*f*) 'Happiness, like youth, has here no second spring.'

(*g*) 'The most may err as grossly as the few.'

(*h*) 'There are few minds but might furnish some instruction.'

17. Analyse logically the following sentences :—

(*a*) 'Some must work or all would die.'

(*b*) 'All can hear, none heed, his neighbour's call.'

(*c*) 'If a University gives stimulative teaching, it does everything; if it fails to do this, it does nothing.'

(*d*) 'All these men were my friends.'

(*e*) 'All those letters were not writ to all.'

(*f*) 'Some are slaves everywhere, others nowhere.'

(*g*) 'Natural productions are not all equally perfect.'

(*h*) 'All would be losers were all to work for all.'

(*i*) 'We all are frail.'

(*j*) 'All men are not born to reign.'

18. Express in logical form, giving reasons :—

(*a*) 'Not every brilliant speech is a proof of wisdom.'

(*b*) 'None of those examined failed to pass.'

(*c*) 'I am sure there is not a man of honour who would not shrink from such humiliation.'

(*d*) 'There is one thing only which gathers people into seditious communities, and that is oppression.'

(*e*) 'Both theory and practice prove the overwhelming importance in modern war of guns of large calibre.'

(*f*) 'The enemy resisted desperately, but unavailingly.'

(*g*) 'Every man of military age, unless he be exempted as physically unfit or as indispensable to some work of national importance, is sent into training.'

(*h*) 'Nothing could exceed the ludicrous effect of arrogant confidence on the part of the senior, if we except the baseness and degradation which are thus, by his misconduct, perpetually inculcated upon, and cultivated in, the minds of youth.'

CHAPTER V.

[N.B. *In dealing with exercises on actual sentences, substitute letters for the terms, then determine the relations, and translate back into language.*]

1. What is meant in Logic by Laws of Thought? Show the relation of the Laws of Thought to the inferences based upon the Square of Opposition.

2. 'Two propositions may be formally consistent, but neither inferable from the other.' Explain this and give examples.

3. Explain and illustrate: 'Of propositions that are logically opposite, those related in subalternation may both be true, and may both be false; those related as contradictories may not both be true, and may not both be false; those related as contraries may not both be true, and may both be false; and those related as sub-contraries may both be true, and may not both be false.'

4. Show how the words Contrary and Contradictory are technically used in application (*a*) to Terms, (*b*) to Propositions.

Illustrate your explanation by reference to the terms *Useful*, *Useless*, *Harmful*, *Harmless*, and to propositions in which these terms occur as predicates.

5. (*a*) 'There is no S that is P.'

(*b*) 'There is at least one S that is not P.'

(*c*) 'There is at least one S that is P.'

(*d*) 'There is no S that is not P.'

Find the relations of opposition between the above four propositions; and determine, in the case of each proposition, whether the term S and whether the term P is distributed or undistributed.

6. By what tests do you decide whether one proposition is the logical contradictory of another? Use the following propositions and their contradictories to illustrate your answer:—

(*a*) 'Not one of the enemy escaped.'

(*b*) 'Anyone but a fool would see this.'

(*c*) 'There are frequent contradictions in the daily newspapers.'

(*d*) 'Only Conservatives voted for the bill.'

7. What relation exists between the proposition 'Some traders visit that locality' and—

(*a*) 'Some visitors of that locality are not persons other than traders';

(*b*) 'All traders visit that locality';

(*c*) 'Some traders are not persons who do not visit that locality'?

8. It being given that the proposition 'All House-flies are Insects' is true, make a list of all the propositions which we thence know to be false.

It being given that the proposition 'All Spiders are Insects' is false, make a list of all the propositions which we thence conclude to be true.

9. Point out the logical relation, if any, that exists between the proposition 'The cold always kills these plants' and—

(*a*) 'What fails to kill these plants is never the cold';

(*b*) 'Sometimes the death of these plants is caused by the cold';

(*c*) 'Sometimes the cold does not kill these plants';

(*d*) 'The heat preserves the life of these plants';

(*e*) 'The cold never fails to kill these plants.'

10. In what way is the proposition 'No statesman has dealt adequately with that question' related to—

(*a*) 'One statesman at least has dealt adequately with that question';

(*b*) 'Some persons who have not dealt adequately with that question are statesmen';

(*c*) 'All statesmen have dealt adequately with that question';

(*d*) 'All statesmen have not dealt adequately with that question';

(*e*) 'Some who are not statesmen have dealt adequately with that question'?

Define each of the relationships you mention.

11. What meaning is it best to assign to 'some'? Give your reasons. If 'some' were taken to mean 'some only,' how would an **I** proposition be related to **A** and **O**? And how, in that case, would you contradict the statement that 'some swans are white'?

12. Define a Hypothetical Proposition. Take various hypothetical propositions and show how they are contradicted.

13. When is a term said to be distributed? Which terms are distributed in the following propositions:—

(*a*) 'That is a mistake';

(*b*) 'Ignorance is not always bliss';

(*c*) 'Adjectives have no meaning except in connexion with a substantive';

(*d*) 'The order and connexion of ideas is the same as the order and connexion of things'?

14. Show that the processes of Obversion are justified by reference to the Laws of Thought. Derive as many immediate inferences as you can from the proposition 'Whenever *A* is found to be *B*, *C* is present.'

15. Define Contraposition. Give the contrapositive of :—
(*a*) 'That is fallacious';
(*b*) 'Juries consist of twelve men';
(*c*) 'To forgive is divine.'

16. Give the (*a*) contradictory, (*b*) obverse, (*c*) contradictory of the obverse, (*d*) obverse of the contradictory, of 'Things are not what they seem.'

17. What immediate inferences can be drawn from a particular negative statement? State all the immediate inferences you can from the following propositions :—
(*a*) 'An article does not always rise in price on being taxed.'
(*b*) 'A triangle cannot have more than one obtuse angle.'

18. Give, where possible, the contrary, the contrapositive, and the obverted converse of :—
(*a*) 'All the angles of the triangle ABC are less than 90 degrees.
(*b*) 'Health cannot be long maintained without exercise.'
(*c*) 'Some mute inglorious Milton here may rest.'
(*d*) 'So careful of the type she seems.'
(*e*) 'All men are not naturally musical.'
(*f*) 'None came to grief except those who paid no heed to the warning.'

19. Give, where possible, the contradictory, the obverse, and the contrapositive of the following :—
(*a*) 'Some who were present failed to understand what was said.'
(*b*) 'No persons will be admitted without special permission.'
(*c*) 'Lycidas is dead.'
(*d*) 'Never when *A* is *B* is it the case that *C* is *D*.'
(*e*) 'All men are sometimes discontented.'
(*f*) 'Hardly anybody escapes altogether the pitfalls of metaphor.'

20. Express the following statements in logical form, and add the contradictory, the obverse, and the converse (if any) :—
(*a*) 'Candidates who fail are either lazy or incompetent.'

(*b*) 'All members of Convocation, except the women, have a vote.

(*c*) 'Only members of Convocation have a vote.'

(*d*) 'Few people can describe adequately what they see.'

(*e*) 'One always has generous intentions when one is poor.'

(*f*) 'Hardships are not infrequently blessings in disguise.'

CHAPTER VI.

1. "Our method . . . of discovering the sciences is one which leaves not much to acumen and strength of wit, but nearly levels all wits and intellects." (*Bacon.*)

"Method implies a progressive transition. . . . The term, method, cannot, therefore, otherwise than by abuse, be applied to a mere dead arrangement, containing in itself no principle of progression." (*Coleridge.*)

Comment on these two statements, and determine which most truly expresses the nature of method.

2. What are the principles observed in methodical thought? Illustrate by an example.

3. 'All method is related to system.' Explain and illustrate.

4. Criticise: 'This may be an inference to you, but it has long ceased to be one with me, for I worked it out years ago.'

5. Criticise: 'Inference is impossible. For if it goes beyond its premises it is invalid, and if it does not, there is no advance in knowledge.'

6. Distinguish between the Matter and the Form of Thought.

Consider the statement that all deductive logic is purely formal and all inductive logic is material.

7. Distinguish between inductive and deductive inference.

8. Make clear that both Analysis and Synthesis are involved in both Induction and Deduction.

9. Explain and illustrate the different meanings of the word 'Law' in the expressions 'Law of Nature,' 'the Law of England,' 'the Laws of Thought.'

10. Explain and give examples of (*a*) Axioms, (*b*) Empirical Laws.

11. Comment on : "It is no doubt true, if a law be universal, it will be confirmed by all our experiments ; but it hardly follows that, because all our experiments have failed to detect an exception, it is true universally." (*L. Stephen.*)

CHAPTER VII.

1. Write explanatory notes on Fallacy, Sophism, Paradox.

2. Discuss : 'The falsity of the premises does not prove the falsity of the conclusion, nor does the truth of the conclusion prove the truth of the premises.'

3. Consider the following :—

Counsel : 'There was no written agreement for the sale of this carpet ? '

Plaintiff : 'Well, you don't have a written agreement when you buy a loaf.'

Counsel : 'You don't cover a floor with a loaf.'

Plaintiff : 'Neither do you eat a carpet.'

4. What fallacies are illustrated in the following extracts from *The Noodle's Oration* (*Sydney Smith*) ?—

(*a*) "Was the honourable gentleman (let me ask him) always of this way of thinking ? Do I not remember when he was the advocate in this House of very opposite opinions ? I do not quarrel with his present sentiments, Sir, but I declare very frankly I do not like the party with which he acts. If his motives were as pure as possible, they cannot but suffer contamination from those with whom he is politically associated."

(*b*) "Nobody is more conscious than I am of the splendid abilities of the honourable mover, but I tell him at once, his scheme is too good to be practicable. It savours of Utopia. It looks well in theory, but it won't do in practice."

(*c*) "Instead of reforming others—instead of reforming the State, the Constitution, and everything that is most excellent, let each man reform himself ! Let him look at home, he will find

there enough to do, without looking abroad, and aiming at what is out of his power."

5. Examine critically the claims to be axiomatic of—

(*a*) 'Minorities must always suffer.'

(*b*) 'It is illogical to treat men differently on account of the colour of their skins.'

(*c*) 'All men are born and remain free and equal in respect of rights.'

6. 'The force of Analogy does not depend on the *amount* of resemblance, but on the *character* of the resemblance.'

Discuss this view, and analyse in accordance with your criticism this argument: 'The ruler of the State being the head of the social family has the duties as well as the rights of a parent.'

7. Annotate :—

'Analogy may be perfect where there is no resemblance.'

'Analogies are used in aid of conviction; metaphors as means of illustration.'

8. Consider the value of the analogy implied in the following query of Berkeley :—

"Whether the mind, like the soil, doth not by disuse grow stiff; and whether reasoning and study be not like stirring and dividing the glebe?"

9. Examine the following (from *Thomas Fuller*) :—

(*a*) "The best is, that unconscionable liars, though they most hurt themselves, do the least harm others, whose loud ones are both the poison and the antidote, seeing no wise man will believe them. Small grit and gravel may choke a man; but that stone can never stop his throat which cannot enter into his mouth."

(*b*) "The judge may be deceived by false evidence. But blame not the hand of the dial, if it points at a false hour, when the fault is in the wheels of the clock which direct it, and are out of frame."

(*c*) "Hence, as hills, the higher, the barrener; so men commonly, the wealthier, the worse."

10. Examine critically :—

(*a*) 'Knowledge of facts is more valuable than knowledge of words; therefore, the study of science improves the mind more than the study of literature.'

(*b*) 'Your opinions are sometimes erroneous, so you should always distrust your judgement.'

(*c*) 'You never give an opinion unless you believe yourself to be right; therefore, you must consider yourself infallible.'

11. What fallacy did Dr. Johnson commit when he said: "The truth is, that luxury produces much good. A man gives half-a-guinea for a dish of green peas; how much gardening does this occasion!"

12. "Some years ago a man was tried for stealing a ham, and was acquitted upon the ground that what was proved against him was that he had stolen a portion of a ham." (*De Morgan.*)

What fallacy was avoided by the verdict?

13. Examine: 'If it be true that all men and women are players, it follows that all players are men and women, and that, therefore, a great player ought to be a great man.'

14. What fallacy does Sir A. W. Ward impute to Macaulay in the following passage from the *Cambridge History of English Literature?*

"It cannot be denied that, while, in this never-ending process of research, like a great advocate gifted with the faculty of sweeping everything into his net except what he has no desire to find there, he never lost sight of facts that would be of use and of value to him, he, on occasion, omitted to bring in facts adverse to his conclusions."

15. Examine the following—

Counsel for the Defence: 'It is said the prisoner had access to the till; but so had several others: that he spent money freely soon after the theft was committed; but he may have received funds from a legacy, from winning a bet, or from good fortune at cards: that he has been proved guilty of theft on a previous occasion; but remember the proverb, "once bitten, twice shy": that he was the last to leave the shop the evening before the theft was discovered; but it is not proved that the theft was committed at that time. I submit that none of these facts proves him to be guilty.'

16. Mention with a short explanation of them some of the fallacies that are incidental to the process of Induction.

CHAPTER VIII.

1. Show that all observation involves selection, judgement, and inference.

2. Discuss the nature of accurate and adequate observation, and indicate, with illustrations, the mistakes most easily made.

3. Discuss the nature of scientific instruments, and explain: "Skill in modern laboratory work is as far out of the reach of the untaught as performance on a musical instrument." (*Allbutt.*)

4. Distinguish between Observation and Experiment. To what class of investigation is each applicable? Is it correct to speak of 'Fallacies of non-Observation'?

5. Why is it that sciences that are mainly dependent on Observation are at a great disadvantage compared with those in which Experiment can be largely employed? Give instances of the difficulty that may arise in deciding where to draw the line between Observation and Inference.

6. Comment on: 'It is possible to work several hours a day for years in a scientific laboratory without being trained as an experimenter.'

CHAPTER IX.

1. State and illustrate the chief classes of relation between facts established and used by thought.

2. Explain what is meant by generalisation, and indicate, with examples, the limits within which it is legitimate.

3. What do you understand by 'scientific' thought and knowledge?

4. Induction has been described as a procedure from the known to the unknown. Is that description adequate? If not, can you offer a better description of Induction?

5. Describe and exemplify the three stages in the Deductive Method of Induction.

6. Define Hypothesis, and give some tests for judging the value of a hypothesis. How does Euclid employ them?

7. Examine: 'A facility in framing hypotheses, if attended with an equal facility in laying them aside when they have served their turn, is one of the most valuable qualities a philosopher can possess.'

8. State as clearly as you can the nature of an analogical argument. Give an instance, taken from natural science, of such an argument.

9. Explain: 'In generalisation the resemblances have great extension and little intension, whereas in analogy we rely upon the great intension, the extension being of small account.'

10. What is meant by the statement that in analogical arguments we argue *explicitly* from Particular to Particular, but *implicitly* from Particular to Universal? In the course of your answer give an example of an argument by Analogy.

11. What do you understand by a 'working hypothesis'? Give examples.

12. What is meant by 'Imperfect Induction'? In particular consider whether Imperfect Induction involves a process of Deduction.

13. Give a critical account of Induction by Simple Enumeration, and explain precisely wherein lies the value of multiplying observations in the course of an inductive inquiry.

14. In what parts of the inductive process is *the mental initiative* especially valuable?

15. If you received a number of anonymous letters what steps would you take to discover who sent them? What evidence would you regard as sufficient to establish the guilt of a person you suspected?

16. Cicero relates that a prisoner, "having spent his entire life in a dark dungeon, and knowing the light of day only from a single ray which passed through a fissure in the wall, inferred that if the wall were removed, as the fissure would no longer exist, all light would be excluded." (*Lecky.*)

Analyse the inference and show clearly where it failed.

17. Test the validity of the following argument: "Jeffrey [the dwarf] was born in the parish of Okeham in this county, where his father was a very proper man, broad shouldered and chested, though his son never arrived at a full ell in stature.

It seems that families sometimes are chequered, as in brains so in bulk, that no certainty can be concluded from such alternations." (*Fuller.*)

18. Discuss critically: "The Rev. Hilderic Friend vouches for the genuineness of the following story: . . . 'In the village of S——, near Hastings, there lived a couple who had named their first-born girl Helen. The child sickened and died, and when another daughter was born, she was named after her dead sister. But she also died, and on the birth of a third daughter the cherished name was repeated. This third Helen died, "and no wonder," the neighbours said; "it was because the parents had used the first child's name for the others."'" (*Clodd.*)

19. "I am a Jew. Hath not a Jew eyes? hath not a Jew hands, organs, dimensions, senses, affections, passions? fed with the same food, hurt with the same weapons, subject to the same diseases, healed by the same means, warmed and cooled by the same winter and summer, as a Christian is? If you prick us, do we not bleed? if you tickle us, do we not laugh? if you poison us, do we not die? and if you wrong us, shall we not revenge? if we are like you in the rest, we will resemble you in that." (*Shakespeare: Merchant of Venice.*)

Examine this reasoning logically.

20. Consider the nature and force of the following inference by Sherlock Holmes—

"'I perceive that you have been unwell lately. Summer colds are always a little trying.' . . .

'How did you know it?' . . .

'Your slippers are new,' he said. 'You could not have had them more than a few weeks. The soles which you are at this moment presenting to me are slightly scorched. For a moment I thought they might have got wet and been burned in the drying. But near the instep there is a small circular wafer of paper with the shopman's hieroglyphics upon it. Damp would of course have removed

this. You had then been sitting with your feet out-stretched to the fire, which a man would hardly do even in so wet a June as this if he were in his full health.'" (*A. Conan Doyle : The Stockbroker's Clerk.*)

21. Comment on the following: "The inductive method has been practised ever since the beginning of the world by every human being. It is constantly practised by the most ignorant clown, by the most thoughtless schoolboy, by the very child at the breast. That method leads the clown to the conclusion that if he sows barley he shall not reap wheat. By that method the schoolboy learns that a cloudy day is the best for catching trout. The very infant, we imagine, is led by induction to expect milk from his mother or nurse, and none from his father." (*Macaulay: Essay on Bacon.*)

22. Try to solve the problem in the passage marked (*a*), without looking at that marked (*b*). Then compare your solution with that given in (*b*).

(*a*) "Suppose a number of boys are in a field playing football, whose superfluous garments are lying about everywhere in heaps; and suppose you want, for some reason, to find out in what order the boys arrived on the ground. How would you set about the business?"

(*b*) "Surely you would go to one of the heaps of discarded clothes, and take note of the fact that this boy's jacket lay under that boy's waistcoat. Moving on to other heaps you might discover that in some cases a boy had thrown down his hat on one heap, his tie on another, and so on. This would help you all the more to make out the general series of arrivals. Yes, but what if some of the heaps showed signs of having been upset? Well, you must make allowances for these disturbances in your calculations. Of course, if some one had deliberately made hay with the lot, you would be nonplussed. The chances are, however, that given enough heaps of clothes, and bar intentional and systematic wrecking of them, you would be able to make out pretty well which boy preceded which; though you could hardly go on to say with any precision whether Tom preceded Dick by half a minute or half an hour." (*Marett: Anthropology.*)

CHAPTER X.

1. Comment on: 'The use of classification is to fix the attention upon the distinctions which exist among things; and that is the best classification which is founded upon the most important distinctions, whatever may be the facilities which it may afford of ticketing and arranging the different objects which exist in nature.'

2. Explain briefly the relation in which Classification stands to Induction. Give examples in illustration of your answer.

3. State carefully wherein the utility of Logic consists. If Logic is defined as the *Science of Reasoning* ought the logician to take account of Classification?

4. State the rules of logical division, showing by examples the meaning of (*a*) *fundamentum divisionis*, (*b*) exhaustive division, (*c*) cross division.

5. What is meant by a Natural Classification? Illustrate your answer by means of a classification (*a*) of Terms, (*b*) of Propositions.

6. Distinguish briefly between a classification for a special purpose and a classification for general purposes. 'All classification implies abstraction.' Explain this statement.

7. What is a Descriptive Terminology? Mention any sciences which suffer from defectiveness in this respect.

8. When is a division inadequate? When indistinct? And when not arranged according to proximate parts?

9. What are the characteristics of a good logical Division? Criticise the following Divisions:—

(*a*) Pictures into landscapes, sea-pieces, portraits, and water-colours;

(*b*) Ships into sailing vessels, liners, and men-of-war;

(*c*) English coins into sovereigns, half-sovereigns, silver, and copper;

(*d*) Matriculation candidates into those who pass well, those who fail badly, those who are near the line between passes and failures, those who pass in Logic, and those who fail in Logic.

10. What is the purpose of *Definition*, and what are the chief ways in which a Definition may fail in its purpose?

Explain, with examples, the special advantages secured by defining a species *per genus et differentiam*.

11. In what respects do the definitions that occur in science differ from, or fall short of, what is signified by definition in its technical sense?

12. Discuss the place of Definition in Formal Logic; and say how definitions are obtained and are logically tested.

13. Explain: 'Modifications of our definitions accompany the progress of knowledge.' How can this be if Definition is the exposition of the connotation of a term, and connotation consists of those attributes of which the name is a mark in the general mind?

14. Illustrate: 'The inductive method of definition collects and compares instances, as varied as possible, of the application of the term to be defined, selects their common attributes, reduces these to the simplest expression, and arranges them in appropriate order.

The method of determination refers the term to a higher class, determines by successive addition of marks its cognate genera, especially its proximum genus, and then adds the marks that distinguish it from its co-ordinates.'

15. State the distinction between real and nominal definitions.

Annotate: 'A definition expresses the analysis of the connotation of a term, and a division expresses the analysis of its denotation;—the former analyses the metaphysical whole, or whole of comprehension, into its parts, the attributes contained in it,—the latter analyses the logical whole, or whole of extension, into its parts, the objects contained under it;—the former states what terms mean, and thus makes symbolical cognitions clear and full,—the latter states what terms name, and makes knowledge complete and distinct;—definitions and divisions are connected with the art of logic, rather than with the science, and are not formally obtained or formally verified.'

16. 'Definition is formal and direct; Division is by comparison material and indirect.' By reference to examples bring out the meaning of this statement.

17. Discuss: 'Definition is by no means an arbitrary process. Can the same be said of Division?

18. Consider whether the following are good definitions:—

(*a*) 'An oblong is a figure whose opposite sides are parallel, and whose angles are right angles';

(*b*) 'The Pygmies are the inhabitants of the great forest of Central Africa';

(*c*) 'A woman is a rational being.'

19. How are definitions obtained? Decide which of the following things is to be considered a house, and why—namely, stables, cowhouses, conservatories, sheds, lighthouses, tents, caravans, hulks, sentry-boxes, ice-houses, summer-houses, and parish pounds.

20. Give a list of the Predicables, and bring out clearly the meaning of each in modern logic.

21. Explain what is meant by Inference. Mention some of its *propria* and separable accidents.

22. Distinguish between general and not general knowledge. Which of the Predicables have reference to knowledge that is general?

23. Explain the relations between Genus, Differentia, and Species; and illustrate the principle of defining a species *per proximum genus et differentiam* by giving two correct definitions of the species *square*.

Give also examples of faulty definitions which do not violate the above principle, pointing out the nature of the faults in each.

24. Give the proximate genus for the following species:—Mass, triangle, plant, hound, monarchy, science. (*Jevons.*)

25. In what respects are the following definitions, or some of them, defective?

(*a*) 'Logic is a guide to correct reasoning.'

(*b*) 'Logic is the art of expressing thoughts in correct language.'

(*c*) 'Logic is a mental science.'

(*d*) 'Logic is the science of the regulative laws of human thought.' (*Jevons.*)

CHAPTER XI.

1. Trace how an intelligent study of logic illustrates the development of theoretical out of practical knowledge.

2. What do you understand by a Law of Nature? Show carefully by means of an example the way in which Induction and Deduction combine in establishing such a law.

3. What do you understand by the Law of Universal Causation? Distinguish with examples between *exciting* and *predisposing* causes.

4. Distinguish between the popular and the logical use of the term 'Cause.' Give an instance of a cause in the logical sense of the term.

5. Distinguish between *alternative* and *concurrent* causes. Is the same effect ever really due to different causes? Illustrate.

6. Define the terms *Uniformity* and *Cause*; and give examples showing the distinction between *Causal Uniformities* and Uniformties of *other* kinds.

7. Distinguish between an *antecedent*, a *condition*, and the *cause*, of an event. What is meant when it is said that the cause of an effect is discovered by a process of elimination?

8. What is a *hypothesis*? How is a hypothesis verified? Which of the 'empirical methods' is fruitful in hypotheses? Which might usefully be employed for verification of a hypothesis?

9. What is the Canon of the Method of Residues? Give an instance of the employment of this Method.

10. What is the function of the so-called Inductive Methods? Contrast the Method of Difference with the Method of Agreement.

11. Discuss this criticism of the Method of Difference: 'If we did know that A was the cause, why resort to the method at all? If we did not know this, how did we first hit upon A out of all the innumerable antecedents, in order to exclude this one from the second set or experiment?'

12. Illustrate the Method of Concomitant Variations as employed in scientific inquiry.

Explain how this method could be treated as an extension of the Method of Difference.

13. By reference to scientific instances of their application bring out the difference between the Method of Concomitant Variations and the Method of Residues. Why have these Methods been said to be forms of the Method of Difference?

14. Explain Mill's Method of Agreement and Joint Method of Agreement and Difference. In what way does the application of the latter Method avoid the imperfection of the former?
[N.B. The 'Joint Method' is Mill's name for the Method of Agreement in Presence and Absence.]

15. What is the difference between 'Inductio per simplicem enumerationem' and the Inductive Methods of Agreement and Difference?

16. Employ literal symbols to make clear the relation that exists between the Joint Method of Agreement and Difference and (*a*) the Method of Agreement, (*b*) the Method of Difference. Which of these methods can be employed when no appeal can be made to experiment?

17. Carefully describe a scientific instance of inductive inference. Does every inductive inference involve a hypothesis?

18. Show the different methods which might be used to establish the fact that the volume of a body depends upon its temperature.

19. How would you set out to discover the relations between, say, temperature and volume?
Explain carefully the logical character of the methods you propose to follow.

20. Examine :—

(*a*) 'By the needle you shall draw the thread, and by that which is past see how that which is to come will be drawn on.'

(*b*) 'For when one's proofs are aptly chosen,
Four are as valid as four dozen.'

21. Mention various applications of the term *induction*.

22. Examine this reasoning, and determine the Method (or Methods) to which it conforms :—

'If we could find two Unions, identical in population, in the proportion of poor, in rents, in the reluctance of the people to accept relief, one with little out-door relief, the other with much out-door relief, we might infer that, if there was a difference in the number of paupers, the difference would probably be due to the method of administration, especially if no other reason could be assigned for the excess of pauperism.'

23. Examine the following statements :—

(*a*) 'That the sun rose yesterday is no proof that he will rise to-morrow.'

(*b*) 'It is the stars,
The stars above us, govern our conditions,
Else one selfmate and mate could not beget
Such different issues.'

(*c*) 'A cause is more than an antecedent and is other than a reason.'

24. "The place of a planet at a given time is calculated by the law of gravitation ; if it is half a second wrong, the fault is in the instrument, the observer, the clock, or the law ; now, the more observations are made, the more of this fault is brought home to the instrument, the observer, and the clock." (*W. K. Clifford.*)

Explain this statement, and bring out its implications.

25. What conclusion can be drawn from the following passage, and by what methods ?

"In all unhealthy countries the greatest risk [of fever] is run by sleeping on shore. Is this owing to the state of the body during sleep, or to a greater abundance of miasma at such times ? It appears certain that those who stay on board a vessel, though anchored at only a short distance from the coast, generally suffer less than those actually on shore." (*Darwin : Journal of Voyage of H.M.S Beagle.*)

26. Analyse the reasoning in the following passage from Lord Macaulay's speech on National Education (1847), and estimate its force : "I say then, Sir, that, if the science of Government be an experimental science, this question is decided. We are in a condition

to perform the inductive process according to the rules laid down in the *Novum Organum.* We have two nations closely connected, inhabiting the same island, sprung from the same blood, speaking the same language, governed by the same Sovereign and the same Legislature, holding essentially the same religious faith, having the same allies and the same enemies. Of these two nations one was, a hundred and fifty years ago, as respects opulence and civilisation, in the highest rank among European communities, the other in the lowest rank. The opulent and highly civilised nation leaves the education of the people to free competition. In the poor and half barbarous nation the education of the people is undertaken by the State. The result is that the first are last and the last first. The common people of Scotland,—it is vain to disguise the truth,—have passed the common people of England. Free competition, tried with every advantage, has produced effects of which . . . we ought to be ashamed, and which must lower us in the opinion of every intelligent foreigner. State education, tried under every disadvantage, has produced an improvement to which it would be difficult to find a parallel in any age or country. Such an experiment as this would be regarded as conclusive in surgery or chemistry, and ought, I think, to be regarded as equally conclusive in politics."

CHAPTER XII.

1. Arrange the experimental methods in the order of their efficacy, and mention the fallacies incident to each.

2. What is the difference between positive and negative instances? Illustrate your answer by reference to the symbolical representation of the Joint Method of Agreement and Difference.

3. 'No one of the Inductive Methods will logically establish a *general* proposition, and general propositions are the only propositions of value in science.' Discuss this theory, and exemplify your own view by reference to the Method of Difference.

4. Point out the part that Deduction plays in inductive inquiries.

5. Why is the process of inductive discovery incomplete without Verification? Mention at least two ways in which Verification may take place. Give examples.

6. Point out the relation that exists between Hypothesis and Verification. Mention some of the forms that each of these processes may take.

7. What two moods of Hypothetical Syllogism are in common use? and what are the fallacies which are committed by their misuse? Give examples.

8. Define and give examples of a Constructive and a Destructive Hypothetical Syllogism.

9. Distinguish the *modus tollens* from the *modus tollendo ponens*. Is the conclusion in the former syllogism necessarily a negative proposition

10. Comment on: "Auguste Comte said that 'Prevision is the test of true theory.' I should say that it is *one test* of true theory, and that which is most likely to strike the public attention. Coincidence with fact is the test of true theory, but when the result of theory is announced beforehand, there can be no doubt as to the unprejudiced spirit in which the theorist interprets the results of his own theory." (*Jevons*.)

11. What is meant by a 'probable' conclusion? Discuss Bishop Butler's saying: 'Probability is the guide of life.'

12. Distinguish between Evidence and Proof. Can either—or both—be strengthened? If so, how?

13. What is the nature of a dilemma, and what is meant by formally 'rebutting' one? In what other ways may dilemmatic arguments be met? Give examples.

14. The following method of reasoning has been sometimes adopted to establish the truth of Newton's laws of motion:—

'The times of the eclipses are calculated by assuming that Newton's laws are correct. Now the calculated times agree with the observed times; therefore the laws are true.'

Give a *formal* statement of this argument, and say whether you consider it correct.

15. How can a decision be reached between two rival hypotheses? Give an example.

16. Criticise the following arguments :—

(*a*) 'Since there are so many people on the verge of starvation, it would be well for the country to give up producing luxuries, and to produce only necessaries.'

(*b*) 'The athlete should bring to bear all the attentive force he can upon his muscular movements, for concentration of attention is most useful in enabling us to perform our work.'

(*c*) 'He speaks the truth, and a man who speaks the truth is always worth hearing.'

(*d*) 'If Mr. *A*. is not in time, he will not be able to perform that ceremony; but we know that he always is in time, and, therefore, he will be able to perform the ceremony.'

(*e*) 'If we cannot go by rail we must go by car; it is clear that we must go by car, and, therefore, we cannot go by rail.'

(*f*) 'Darwin must have been very unhappy. For he said that he would feel very happy if he had only to observe, and not to write; and we know, of course, that he wrote many books.'

(*g*) 'If observation and constructive imagination are essential to sympathy, Mr. *B*. has the sympathetic disposition, since he possesses those capabilities in an eminent degree.'

(*h*) 'If the French exchange had been against England, Mr. *X*. would have lost by that transaction; but Mr. *X*. has lost by that transaction; therefore the French exchange must have been against England.'

17. Put into logical form and criticise the following arguments :—

(*a*) 'If one of the Powers increases its naval force it follows that it will be in the interests of the other Powers to increase theirs.'

(*b*) 'Granted that the laws which govern the winds are once discovered, aerial navigation will be practically free from danger.'

(*c*) 'You had better not write a book on that subject, for those who agree with you do not want to know any more of the subject, and those who do not agree with you will not read what you have written.'

18. Analyse the following arguments and determine their worth :—

(*a*) 'If he had to wait at the station, either the train was late or his watch had been gaining; it was found that his watch had been gaining, and, therefore, the train was not late.'

(*b*) 'Canada needs no navy. For as long as the United States are friendly towards her, no navy is necessary; and if ever the United States should decide to invade Canada, no navy could prevent them.

(*c*) 'This author is certainly confused. If I understand his book rightly, he is confused in his thinking, and if I do not understand it, then he is confused in his writing.'

(*d*) 'As the travellers have arrived, and if it had gone well with them they would have arrived, it is clear that no accident can have happened to them.'

(*e*) 'Agriculturists must be heavy losers, for, if the summer is dry, they lose through the withering of the crops, and, if the summer is wet, they lose because the crops do not ripen.'

(*f*) 'If the fifth proposition of Euclid is not true it is not worth the labour expended upon it; but it is true, and, therefore, labour expended upon it is not wasted.'

(*g*) 'The coal-tax must be right; because, if it raises prices, it benefits the producers; and, if not, it does no harm to the consumers.'

(*h*) 'Lying is the result either of vanity or of cowardice; hence this extravagant boaster must be a brave fellow.'

19. Illustrate with examples: 'A dilemma is often fallacious because one of the possible alternatives is omitted.'

20. Examine these inferences, and name them :—

(*a*) 'You should be women,
And yet your beards forbid me to interpret
That you are so.'

(*b*) 'If you had been an Athenian and I a Seriphian, neither of us would have been famous; but I am famous.'

(*c*) 'Men must be governed by force or by reason; but not by force, for they will rebel; nor by reason, for few listen to it: there fore it is impossible to govern them.'

(*d*) 'If powers divine
Behold our human actions, as they do,
I doubt not then but innocence shall make
False accusation blush.'

(*e*) 'Then let us say, you are sad,
Because you are not merry; and 'twere as easy
For you, to laugh, and leap, and say, you are merry,
Because you are not sad.'

(*f*) "*Dr. Johnson.* 'No honest man could be a Deist, for no man could be so after a fair examination of the proofs of Christianity.' I named Hume. *Johnson.* 'No, sir; Hume owned to a clergyman in the bishoprick of Durham that he had never read the New Testament with attention.'"

(*g*) 'I could teach you
How to choose right, but I am then forsworn;
So will I never be: so may you miss me:
But if you do, you'll make me wish a sin,
That I had been forsworn.'

21. Analyse and estimate the value of—

(*a*) "There is no danger; for if what I relate be done well, it is convenient the world should know it; but if it be ill done, all good men will thank me for discovering it; and they themselves being seized with shame for what they have done, will do so no more."

(*Erasmus: Colloquies.*)

(*b*) 'If all men were capable of perfection, some would have attained it; as none has done so, none is capable of doing so.'

[*N.B.* (1) In a destructive hypothetical syllogism, the quantity of the consequent of the hypothetical premise and that of the categorical premise must be contrary or contradictory.

(2) The conclusion then justified is the contradictory, *but not the contrary*, of the antecedent of the hypothetical premise.]

CHAPTER XIII.

1. Discuss: 'All sciences tend to become more and more Deductive, they are not, therefore, the less Inductive; every step in the Deduction is still an Induction. The opposition is not between the terms Deductive and Inductive, but between Deductive and Experimental.'

2. Define (*a*) Immediate Inference, (*b*) Mediate Inference. State, giving your reasons, under which heading you would place (1) Contraposition, (2) Hypothetical Syllogisms.

3. What do you understand by a syllogism? How is it that when *M* is predicate in both the premises of a syllogism the major premise of the syllogism must be universal?

4. 'In every syllogism we drop from a complex statement some portion not desired at the moment.' Explain this assertion, and illustrate your answer by reference to the case where both premises are singular propositions.

5. If the premises of a valid syllogism are false, does that make the conclusion false? If the premises are true, must the conclusion be true? Give examples.

6. Trace carefully the connexion that exists between the *Dictum de omni et nullo* and the ordinary rules of the syllogism.

7. If the conclusion of a syllogism be universal, is it possible to have the middle term distributed twice in the premises?

8. Explain, on the most general grounds, why no conclusion can be drawn (*a*) when both premises of a syllogism are negative, (*b*) when the middle term is not distributed in either premise. Illustrate your answer.

9. What principle is appealed to in the rule that no conclusion can be reached from two negative premises? Is it ever possible with the occurrence of a negative particle in both premises to draw a valid conclusion?

10. Can the syllogistic rule concerning the impossibility of drawing a conclusion from two negative premises be regarded as a corollary from the rule concerning the necessity for the distribution once at least of the middle term?

11. Deduce from the six rules of the Syllogism the following propositions :—

(*a*) If one premise is particular, so must be the conclusion.

(*b*) From a particular major and a negative minor nothing can be inferred.

12. From the general rules of the syllogism deduce the rule that two particular premises cannot yield a valid conclusion.

13. Explain : 'Granting that the major premise involves the minor premise and the conclusion, that is just the reason why it is imperative to express them ; the meaning of the syllogism is that it analyses the whole actual thought.'

14. Discuss: 'If we interpret the universality of the syllogistic major premise as depending on a complete enumeration, the inference is at once reduced to a *petitio principii* by the direct affirmation of the conclusion in the major premise.'

15. Examine these statements critically :—

'It must be granted that in every syllogism, considered as an argument to prove the conclusion, there is a *petitio principii*.'

'Unless the establishment of an Universal Proposition requires an explicit and conscious examination of every existing and also of every possible particular instance, no charge of Petitio Principii, or even of vain repetition, can be maintained against the Syllogism.'

16. Examine: 'Syllogism in no way increases knowledge if once the premises are given, which is in perfect agreement with the circumstance that no rational human being thinks in syllogisms.'

17. Invent syllogisms to prove the following conclusions :—

(*a*) 'Private property should be respected in war.'

(*b*) 'No woman ought to be admitted to the franchise.'

(*c*) 'Capital punishment ought to be abolished.'

(*d*) 'Royal parks ought not to be used for political meetings.'

18. Construct syllogisms to express the inferences suggested by the following queries :—

(*a*) "Whether men should not in all things aim at perfection? And, therefore, whether any wise and good man would be against applying remedies [for public ills]? But whether it is not natural to wish for a benevolent physician?"

(*b*) "Whether it be not evident, that not gold but industry causeth a country to flourish?"

(*c*) "Whether temporary servitude would not be the best cure for idleness and beggary?" (*Berkeley: The Querist.*)

CHAPTER XIV.

1. What are the four figures of Syllogism?

Why is it that a universal affirmative conclusion cannot be drawn in any but the First Figure?

2. Show that the special rules of the First Figure (*a*) can be deduced from the general rules of the syllogism, (*b*) are implied in the *dictum de omni et nullo.*

3. Show by reference to the rules of the Categorical Syllogism that (*a*) in the Second Figure the major premise must be universal, (*b*) in the Third Figure the conclusion must be particular.

4. Prove that (*a*) in the Third Figure the minor must be affirmative, (*b*) in the Fourth Figure, if the major is affirmative, the minor must be universal, and (*c*) in the Fourth Figure, if the minor is affirmative, the conclusion must be particular.

5. Prove that (*a*) in the Second Figure one premise must be negative, and (*b*) in the Fourth Figure, if either premise is negative, the major must be universal.

6. Derive from the general rules of the Syllogism the special rules of Figure IV.

7. Explain the distinction between *Mood* and *Figure* in the syllogism.

Why are all moods with an **O** premise excluded from the First and the Fourth Figure, while certain syllogisms with **O** as a premise are admitted in the Second and Third Figures?

8. Show that the mood **EAO** is valid in every figure of the syllogism.

9. Find out, from the general rules of the syllogism, in what figures the moods **AEE** and **IAI** are valid.

10. (*a*) How is it that **EIO** is always valid and **IEO** is never valid, if the difference between them is one of mere order of premises?

(*b*) Determine the rules of the Third Figure, and enumerate fully the characteristics of this figure.

11. (*a*) Given **I** as major premise, determine by the general rules of syllogism the question of mood and figure.

(*b*) If the mnemonic of a valid mood end with *s* construct the syllogism.

12. Mention any peculiarity of the moods **AEO**, **EIO**, **IEO**, and **OAO** respectively. When, and why, do **AA** as premises give (*a*) an **A** conclusion, (*b*) an **I** conclusion only, and (*c*) no conclusion.

13. Determine from the rules of syllogism relating to quality or distribution (without reference to the corollaries relating to *quantity*) the fallacies involved in the following syllogisms :—(i) **AEE** in the First Figure ; (ii) **AAA** in the Second Figure ; (iii) **OAE** in the Third Figure ; (iv) **IEO** in the Fourth Figure.

14. Give some examples of a Strengthened Syllogism. Is **OAE** in the Fourth Figure such a syllogism?

15. If the middle term of a syllogism be twice distributed, in what figures may a conclusion be drawn? What must be the quantity of the conclusion?

16. What is the difference between a Disjunctive and a Categorical Syllogism? Mention with examples the moods that arise in the case of the former.

[*N.B.* When 'Disjunctive Syllogism' is mentioned the reference is probably to the Mixed Disjunctive Syllogism. But, as this is not explicitly stated, it is better to deal with both Mixed and Pure Disjunctive Syllogisms in answering the question.]

17. Distinguish between Direct and Indirect Reduction. Is Reduction an essential part of syllogistic doctrine?

18. State clearly the nature of *reductio ad impossibile*, and examine (1) the validity of the reasoning, and (2) the legitimacy of the reduction.

19. Reduce *Festino*, *Disamis*, *Camenes*, *Fesapo*, directly to the First Figure.

20. Reduce *Cesare*, *Darapti*, *Fresison*, indirectly to the First Figure.

21. Reduce *Cesare*, *Camestres*, *Camenes*, by obversion to *Barbara*.

22. Show that all the moods of the First Figure can be directly or indirectly reduced to Barbara.

23. Demonstrate that a case of the *modus tollens* may be reduced to a case of the *modus ponens*. What fallacies may be committed in arguing from a hypothetical major premise?

24. Define and illustrate *Enthymeme*, *Weakened Syllogism*, *Sorites*. Why does a Sorites exclude an **O** premise?

25. What premises are suppressed in the following? Name figure and mood of each completely-stated syllogism :—

(*a*) 'Whosoever loveth wine shall not be trusted of any man; for he cannot keep a secret.'

(*b*) 'It is dangerous to tell people that the laws are not just; for they only obey laws because they think them just.'

(*c*) 'The science of logic is very useful; it enables us to detect our adversaries' fallacies.'

(*d*) 'I shall not derive my opinions from books, for I have none.' (*Jevons.*)

26. Show that the following single propositions may be regarded as enthymemes :—

(*a*) 'If wishes were horses, beggars would ride.'

(*b*) "If I had read as much as my neighbours, I would have been as ignorant." (*Hobbes.*)

(*c*) 'All law is an abridgement of liberty and consequently of happiness.'

(*d*) 'Thales, being asked what was the most universally enjoyed of all things, answered—Hope; for they have it who have nothing else.' (*Jevons.*)

27. Complete such of the following arguments as may be considered sound but incomplete syllogisms :—

(*a*) 'The people of this country are suffering from famine, and as you are one of the people of the country, you must be suffering from famine.'

(*b*) 'Marcus Aurelius was both a good man and an emperor; hence it follows that emperors may be good men.'

(*c*) 'Nothing which is unattainable without labour is valuable; some knowledge is not attainable with labour, and is therefore valuable.'

(*d*) 'Suicide is not always to be condemned; for it is but voluntary death, and voluntary death has been gladly embraced by many heroes.' (*Jevons.*)

If you reject any as unsound, give your reasons for so doing.

28. State the following arguments in full logical form, and, if any of them be fallacious, expose and name the fallacy :—

(*a*) 'None but Unionists voted for this Bill ; and as no one could have voted who was not in the House at the time, all the Members then in the House must have been Unionists.'

(*b*) 'To get into Parliament a man must be either very wealthy or very wise ; and the Member for X. is very rich.'

(*c*) 'Do not give any assistance to this man who begs from you in the street ; I have investigated many such cases, and they have all proved to be impostures.'

(*d*) 'All metals, it is true, are conductors of electricity ; but then the atmosphere is not a metal, and therefore cannot be a conductor of electricity.'

(*e*) '*A* must be heavier than *B*, because *A* weighs *B* down.'

(*f*) 'Since we know that all the crew are patriotic, and that none of the crew are cowards, we may infer that some cowards are not patriotic.'

29. Examine the validity of the following arguments :—

(*a*) 'All the animals have been moved into their new quarters, and all the animals have taken kindly to the change ; therefore all that have been moved into their new quarters have taken kindly to the change.'

(*b*) 'If all those soldiers endured great hardships, and no persons who endured great hardships ought to go unrewarded, then some, but not necessarily all, who ought to go unrewarded are not those soldiers.'

(*c*) 'Mr. X. is a brave man, and we know that none but the brave deserves the fair.'

(*d*) 'The attempts to discover the North-West Passage were not worth making, for such a passage on account of the ice is useless for commercial purposes.'

(*e*) 'The Members filled the House and they cheered repeatedly ; therefore, all the persons in the House cheered repeatedly.'

(*f*) 'Since some men are born great and some achieve greatness, it follows that none who are born great achieve greatness.'

30. Examine technically the following arguments :—

(*a*) 'A successful novelist must have had much personal experi-

ence ; few novelists have had much personal experience ; therefore, few novelists are successful.'

(*b*) 'Only those novelists who have had much personal experience are successful ; few novelists are successful ; therefore few novelists have had much personal experience.'

(*c*) 'No novelist who has not had much personal experience is successful ; not a few novelists have been successful ; therefore, not a few novelists must have had much personal experience.'

(*d*) 'The love of self is akin to the love of money, for the love of either of these objects is productive of misery.'

(*e*) 'He must be a tariff reformer, for all tariff reformers hold these opinions.'

(*f*) 'That knowledge is not worth having, since it has no practical bearings.'

31. Point out any fallacies there may be in the following inferences :—

(*a*) 'He is not a mathematician, for he is either a mathematician or a logician, and he is undoubtedly a logician.'

(*b*) 'He must possess either perseverance or ability, for all persons who possess either of these qualities succeed, and he has succeeded.'

(*c*) 'Money is useful ; money is wealth ; therefore wealth is useful.'

(*d*) 'All the electors present were in favour of the bill ; some electors were not present : therefore some electors were not in favour of the bill.'

(*e*) 'He is sure to do well, for he has much practice in drawing both immediate and mediate inferences, and no one can expect to do well who has not had such practice.'

(*f*) 'All who attacked the fort were killed or wounded ; no cowards attacked the fort ; therefore, none who were killed or wounded were cowards.'

32. Analyse syllogistically the following reasonings, and determine their validity :—

(*a*) "I always thought him a respectable man—he kept his gig." (*De Morgan.*)

(*b*) "That race must have possessed some of the arts of life, for they came from Asia." *De Morgan.*)

(*c*) "His imbecility of character might have been inferred from his proneness to favourites; for all weak princes have this failing." (*De Morgan.*)

(*d*) "It is a greater fault rigidly to censure, than to commit a small oversight. The one showeth himself man, in mistaking; the other no man, in not pardoning a light mistake." (*Fuller.*)

(*e*) "Harmless mirth is the best cordial against the consumption of the spirits; wherefore jesting is not unlawful if it trespasseth not in quantity, quality, or season." (*Fuller.*)

(*f*) "When all is said, it remains true that there are but three ways of living possible in this world—by working, by robbing, or by begging. To beg is infamous, to rob is criminal. If a man will not work neither shall he eat." (*Froude.*)

33. Analyse the passage from St. Thomas Aquinas quoted on pp. 89-90.

34. Analyse and test: "Every one of [the Utopians] is skilled in their law, for, as it is a very short study, so the plainest meaning of which words are capable is always the sense of their laws. And they argue thus: all laws are promulgated for this end, that every man may know his duty; and therefore the plainest and most obvious sense of the words is that which ought to be put upon them; since a more refined exposition cannot be easily comprehended, and would only serve to make the laws become useless to the greater part of mankind, and especially to those who need most the direction of them: for it is all one, not to make a law at all, or to couch it in such terms that without a quick apprehension, and much study, a man cannot find out the true meaning of it; since the generality of mankind are both so dull, and so much employed in their several trades, that they have neither the leisure nor the capacity requisite for such an enquiry." (*Sir T. More: Utopia.*)

CHAPTER XV.

1. State and illustrate the chief forms of relation that can be combined without the use of a middle term.

2. What is the logical signification of the following italicised words :—

(*a*) *Most* men are of his opinion ;

(*b*) If *any* mistake were made *any* one would notice it ;

(*c*) A *few* days were sufficient for that journey ?

3. Explain : '*All*, *every*, *each*, *either*—these collective adjectives are none of them absolutely incapable of being employed for and instead of any of the rest ; but they have each of them its appropriate and most proper sense.'

4. Analyse : (*a*) 'If betting is as bad as gambling, gambling is no worse than betting.'

(*b*) 'If royalty is the best, tyranny must be the worst of governments, because one is pre-eminence of good, the other of evil.'

5. 'The less it became the farther it went, and the more it contained the narrower it became.' Explain and illustrate the logical relation thus described.

6. 'The argument that " A must be 30 miles from C, because A is 10 miles from B, and B is 20 miles from C " is not logical, and therefore not accurate.'

Comment, making clear the sense in which you understand 'logical' to be used.

7. Explain clearly why *simple conversion* can be applied to the first of the following universal affirmative propositions, but not to the second :—

(1) 'All equilateral triangles have their three angles equal to one another.'

(2) 'All equilateral triangles have the property that a circle can be described so as to pass through their three corners.'

8. Analyse and examine : (*a*) " From foolish opinions comes foolish conduct ; from foolish conduct the severest disaster ; and from the severest disaster the most useful warning. It is from the folly, not from the wisdom, of our ancestors that we have so much to learn." (*Bentham.*)

(*b*) "The liberal reward of labour encourages the industry of the common people. The wages of labour are the encouragement of in-

dustry which, like every other human quality, improves in proportion to the encouragement it receives." (*Adam Smith.*)

CHAPTER XVI.

1. Distinguish the logical from the popular notion of Explanation. What is the difference between Explanation and Justification ?

2. Examine the meaning of Scientific Explanation. Mention its varieties.

3. Explain and illustrate : 'Scientific explanation is often limited by the lawlessness of our ultimate experiences ; the irreducibility and the complexity of our experiences ; and the irreducibility of their laws.'

4. Explain : 'A science is a body of organised laws, or truths abstract, general, definite, reasoned, verified, appropriately classified and properly arranged ; it extends perceptions by inference, achieves quantitative prevision, and systematises knowledge.'

5. Annotate these statements :—

(*a*) 'The method of science cannot anticipate science.

(*b*) 'To find the effect of a given cause, experiment is better than observation ; but to find the cause of a given effect observation is better than experiment.'

(*c*) 'The ideal of science consists, according to J. S. Mill, in the knowledge of the fewest assumptions, from which, if given, the whole course of the world might be inferred.'

6. Explain : "Every phenomenon is known as the sum of a set of relations. The total phenomenon—the attraction of the iron by the magnet—is the effect. The separate factors, the presence of the iron and the magnet, each of which is decomposable into various groups of relations to the perceiving subject, and to each other, are the causes. The same phenomenon can always be resolved into the same causes. If the phenomenon differs, some one or more of the components must differ. In this sense the assertion of the uniformity of causation is resolvable into . . . a statement of the

postulate implied in all reason, and which constitutes the very reasoning process, that we can make identical propositions in identical cases." (*Sir L. Stephen.*)

7. Explain and illustrate : 'Logic investigates the principles of evidence formal and material, and of scientific method. It is a practical science and an art ; it has for its subject-matter thought, inference, evidence,—it is pre-eminently the theory of proof, and is indirectly concerned with the elements, the expression, and the effective employment, of proof.

Logic is divided into Formal Logic or the Logic of Consistency, and Material Logic or the Logic of Experience.'

GLOSSARY.

1. Distinguish between a Real and a Verbal proposition. Construct, where possible, real and verbal propositions with the following subjects : self-love, non-militants, Homer, George Washington, triangle.

2. Write short notes on, and give examples of—

(*a*) An Ampliative Proposition, (*b*) an Exclusive Proposition, (*c*) Infinite or Limitative Proposition, (*d*) Explicative Proposition, (*e*) Exclusive Proposition, (*f*) Remotive Propositions, (*g*) Analytic Proposition, (*h*) Exponible Propositions, (*i*) Equivalent Propositions, *j*) Indesignate Propositions, (*k*) *Fundamentum relationis*, (*l*) Subcontrary Opposition.

3. Define Positive, Negative, and Privative Names. To which of these classes would you assign the terms *Idlers* and *Aliens* ?

4. Write notes on, and give examples of—

(*a*) Categorematic Word, (*b*) Univocal Terms, (*c*) Substantial Terms, (*d*) Distributive use of Terms.

5. Explain the following terms and expressions, as used in Logic : Accident, Denotation, Dichotomy, Analysis.

6. Write brief notes on, and give examples of—(*a*) *Argumentum ad judicium*, (*b*) *Argumentum ad populum*, (*c*) Fallacy of Accident, (*d*) Converse Fallacy of Accident, (*e*) *Petitio principii*, (*f*) *Ignoratio elenchi*, (*g*) Argument from Authority.

7. Write notes on, and give examples of—(*a*) Exhaustive Division, (*b*) Division by Dichotomy, (*c*) *Fundamentum divisionis*, (*d*) Cross Division, (*e*) Metaphysical Division, (*f*) Natural Classification, (*g*) Artificial Classification, (*h*) Definition *per genus et differentiam*, (*i*) Genetic Definition, (*j*) Circular Definition, (*k*) Predicable, (*l*) *Species praedicabilis*, (*m*) Genus and Species, (*n*) Natural kinds, (*o*) *Infima Species*, (*p*) Inseparable Accident.

8. Explain and illustrate the meaning of the following terms: (*a*) *Composition of causes*, (*b*) *Plurality of causes*, (*c*) *Counteracting causes*.

9. Explain by an example in each case the following: (*a*) Compound Effect, (*b*) Reasoning by Analogy, (*c*) Hypothesis, (*d*) Quantitative Induction, (*e*) Empirical Generalisation, (*f*) Derivative Law, (*g*) Empirical Law, (*h*) Crucial Instances, (*i*) Crucial Experiment, (*j*) Consilience of Inductions, (*k*) *A* tends to produce *B*, (*l*) Dilemma, (*m*) Destructive Dilemma, (*n*) Rebuttal of a Dilemma.

10. Mention the Dictum that applies to categorical syllogisms of the First Figure. Show that the conclusions of *Celarent* and *Ferio* are justified by reference to this dictum. Can you give a dictum for the Hypothetico-Categorical Syllogism, and one for the Second Figure of the Categorical Syllogism?

11. Write notes on, and give examples of—(*a*) Perfect Figure, (*b*) *Metathesis praemissarum*, (*c*) Destructive Syllogism, (*d*) *Abscissio infini*, (*e*) *Reductio ad impossibile*, (*f* Strengthened Syllogism, *g*) Weakened Syllogism, (*h*) Hypothetico-categorical Syllogism, (*i*) The limits of Explanation.

12. Justify and illustrate: 'If we obvert the contrapositive of the major of any form of the *Modus Ponens*, we get the corresponding form of the *Modus Tollens*; and the latter can be similarly reduced to the former.'

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[*N.B.—The terms explained in the Glossary are not included in the Index.*]

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